90
Professional paper
Doi 10.61892/STP202601049K
ISSN 2566-4484

Đuro Krnić, University of Novi Sad, djuro.geo@uns.ac.rs
Milan Tadić, University of Novi Sad, tadic.m@hotmail.com
Zoran Sušić, University of Novi Sad, zsusic@uns.ac.rs
Radovan Đurović, University of Montenegro, zlatko1979@yahoo.com
Anastasija Božić, University of Novi Sad, anastasija.bozic@uns.ac.rs

GEODETIC MONITORING OF THE BRIDGE OVER THE DANUBE ON THE BYPASS ROAD AROUND NOVI SAD

Abstract:

The paper describes the phases of bridge construction, geodetic measurements and monitoring procedures during the characteristic phases of construction. In the experimental part of the work, the procedure of geodetic observation of the bridge construction based on the analysis of two measurement epochs is shown. Open-source software Java Applied Geodesy 3D (JAG3D) was used for the processing of deformation measurements with the Karlsruhe method of deformation analysis implemented.

Keywords: engineering geodesy, bridges, deformation analysis, JAG3D, Karlsruhe method.

ГЕОДЕТСКИ МОНИТОРИНГ МОСТА ПРЕКО ДУНАВА НА ОБИЛАЗНИЦИ ОКО НОВОГ САДА

Сажетак:

У раду су описане фазе изградње моста, геодетска мерења и поступци мониторинга током карактеристичних фаза изградње. У експерименталном делу рада приказан је поступак геодетског осматрања конструкције моста на бази анализе две мерне епохе. За потребе обраде деформационих мерења коришћен је софтвер отвореног кода *Java Applied Geodesy 3D* (JAG3D) са имплементираном методом деформационе анализе Karlsruhe.

Кључне речи: инжењерска геодезија, мостови, деформациона анализа, JAG3D, Karlsruhe метода.

1. INTRODUCTION

Monitoring the deformations of engineering structures is one of the key activities in engineering geodesy. From a geodetic perspective, it is essential to provide timely information on the structure's behavior in space and time, with adequate accuracy [1], [2]. As requirements for structural safety, reliability, and durability increase, the role of precise geodetic measurements becomes increasingly important. Based on geodetic measurements, the obtained data must be processed to provide statistically supported conclusions. This is particularly critical for structures in which even small deviations may have constructive and functional significance, such as bridges, dams, tunnels, and high-rise buildings. In this context, deformation monitoring represents a key element of quality assurance during construction and throughout the service life of the facility. The achieved accuracy of deformation monitoring depends on an appropriately designed geodetic network, a suitable observation plan, and stability testing of both the reference points and the object points.

Geodetic works during bridge construction encompass several phases, including the establishment and maintenance of the geodetic control network, setting-out and positional control of structural elements, control measurements during individual construction stages, as well as dedicated measurements for deformation analysis [1], [3], [4], [5], [6], [7]. In modern practice, a geodetic network designed for bridge monitoring, consisting of a reference network and a network of points on the object. The reference network consists of points outside the zone of expected deformations, whereas the points on the object serve as monitoring points for deformation assessment [2], [8]. Bridge construction is complex to monitor from a geodetic monitoring perspective, and measurements of directions, zenith angles and slope lengths in the network are most often performed using high-performance total stations.

The main goal of deformation measurements is to obtain information about the movement of the ground and/or the movement of the object in defines as time intervals. In geodetic practice, this is most commonly achieved by measuring the same quantities in at least two observation epochs. After separate adjustments of each epoch, displacement vectors are formed from the coordinate differences, and their statistical significance is subsequently tested [2], [9], [10]. At the same time, the quality of displacement detection depends on the geometry and stability of the reference frame, the accuracy and reliability of observations, the selected tests and significance level, as well as the appropriate mathematical model of deformation analysis.

In the literature, deformation analysis is often approached through a hierarchy of models, where congruent and kinematic models are considered descriptive, while static and dynamic models are associated with cause-effect relationships [11], [12], [13], [14]. Although the causal interpretation of deformations is increasingly prevalent in engineering practice today, congruent models based on comparing the results of two independent measurement epochs still dominate.

Within the framework of conventional deformation analysis (CDA), a number of procedures have been developed. Among the pioneering approaches, the Pelzer method (Hannover procedure) stands out, while the Karlsruhe method, Caspary method, Delft, Fredericton, Munich and other procedures are widely represented in practice and literature [2], [10], [15]. The choice of method is not trivial, as different approaches to deformation analysis can yield different conclusions about the stability of certain points. That is why modern works often emphasize the need for clear definitions of measurement assumptions and observations, as well as the decision to define the datum of the geodetic network.

With geodetic networks, the relative position of the points is determined by direct measurement, while the problem of datum defects of the geodetic network must be solved to obtain absolute coordinates. In practice, two approaches to defining datum are common, the classic approach and the minimal-trace approach using the cofactor matrix of unknown parameters. Both methods enable adjustment of the free geodetic network. This approach is particularly suitable for monitoring networks because it reduces the dependence of the results on the selection of fixed points. At the same time, stability control of the reference frame is performed before the final interpretation of the displacement [9], [16].

Software tools based on the least squares adjustment (LSA) and formal statistical tests are widely used in engineering practice [16], [17], [18]. The software used in this work is JAG3D, an open-source tool for adjusting geodetic networks. JAG3D enables the combination of hybrid terrestrial observations (height differences, directions, lengths, vertical angles) in a unique mathematical model, as well as the implementation of statistically based deformation analysis.

2. GEODETIC WORKS DURING BRIDGE CONSTRUCTION

Geodetic works are an integral part of the bridge construction process because they ensure the spatial definition of structural elements and control the structure's geometry throughout all phases of the work. Precise geodetic measurements allow the correct transfer of designed elements to the field, control of the executed as-built condition and timely detection of deviations from the design solution.

During the construction of the bridge, geodetic works include the establishment and maintenance of the geodetic control network, the setting-out of the axes and characteristic points of the structure, the control of foundations, piers, span structures, and the monitoring of the object's movement and settlement during certain construction phases. Geodetic works are particularly important for complex bridge structures, such as cable-stayed bridges, where the requirements for accuracy and geometry control are extremely high.

2.1. CONSTRUCTION OF BORED PILES

Before drilling the bored piles, a temporary platform must be built from which the piles will be carried out. The temporary platform is a structure that continues on the temporary bridge to build the foundation and piers of the main bridge structure (Figure 1). The platform is located between the arms of the temporary bridge and is directly statically and functionally connected to the arms. The elements of the temporary construction were marked using Global Navigation Satellite System (GNSS) in the construction site's unique coordinate system.

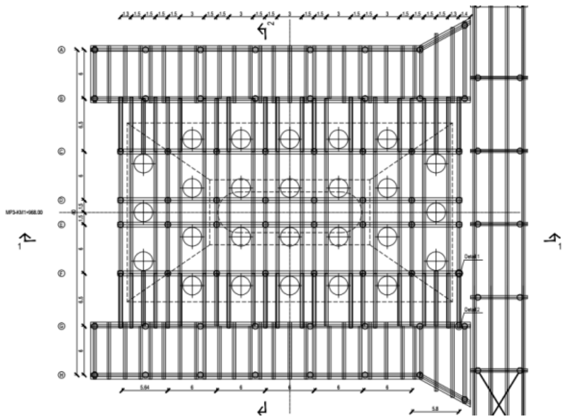

Figure 1. Temporary working platform for pile construction (created by author).

Upon completion of the temporary platform, the setting out of the piles is carried out. Pile setting out is performed using a total station. The accuracy of the setting-out process depends on the quality of the geodetic control network, the number and spatial distribution of orientation points, and the precision of the surveying instrument. Prior to the commencement of drilling, steel casing pipes are installed at the marked pile locations. The position of the pipe is controlled by measuring at least three characteristic points along the circumference of the casing, evenly distributed around its perimeter. Based on the measured data, the center position of the pipe is determined and compared with the designed pile location. After the installation and verification of the pipe position, pile drilling operations commence.

2.2. CONSTRUCTION OF THE COFFERDAM

After the pile drilling phase, the construction of the cofferdam is initiated. A cofferdam is a temporary protective structure that is used during foundation construction in the presence of water or unstable soil. Its main function is to provide a dry and stable working space. Before starting the works, the positions of the cofferdam are precisely marked in accordance with the design, using the established geodetic network of the construction site.

During cofferdam construction, systematic geodetic monitoring of the steel sheet piles and supporting structures is conducted. Before dewatering, geodetic monitoring points are set. After the points are set, geodetic measurements are taken, which represent the zero state. These data serve as a reference basis for the analysis of subsequent movements and deformations of the cofferdam.

Geodetic monitoring is carried out in the phase of water pumping and installation of the supporting structure. Based on repeated measurements, the spatial changes in the positions of the set points are analyzed, and the displacements and deformations of the cofferdam are determined. If the determined displacements exceed the designed or permitted values, it is necessary to suspend further work and implement appropriate technical measures. The layout of the monitoring points is defined in the design documentation and is graphically presented in the following section of the paper (Figure 2).

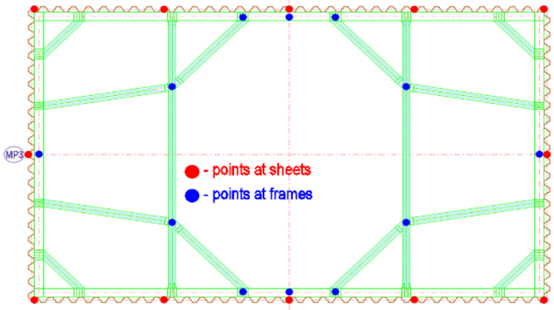

Figure 2. Layout of monitoring points on the sheet piles and supporting structure (created by author).

2.3. FOUNDATION CONSTRUCTION

Before the construction of the foundation, an elevation control of the blinding concrete layer is performed. The blinding concrete represents a thin layer of concrete used for surface levelling, protection of the structural elements, stabilization of the subgrade, and filling of voids. After the execution of the blinding layer, horizontal and vertical control of the reinforcement and formwork of the foundation structure is carried out. Immediately before the commencement of concreting, a final geodetic verification of the formwork position is performed with respect to the design documentation, including the transfer of the designed concrete elevations. Upon completion of concreting and after the concrete has reached the required strength, geodetic measurements of the as-built foundation structure are conducted.

Before proceeding with pier construction, it is necessary to install two temporary monitoring points on the foundation structure. These monitoring points are used to control the settlement of the structure during subsequent construction phases. The elevations of the monitoring points are determined by trigonometric levelling from benchmarks of the site control network. This procedure establishes a reference basis for monitoring vertical displacements and ensures proper continuation of construction works on the bridge pier.

2.4. PIER CONSTRUCTION

The first phase of pier construction involves setting out the designed position of the pier on the completed foundation. This setting out procedure provides the basis for the correct positioning of the reinforcement and formwork of the pier. After the installation of the pier formwork, positional control and verification of the verticality of the formwork are carried out. If any deviations are detected, the required corrections must be indicated and implemented in a timely manner.

Upon completion of the pier construction, an as-built control survey is performed, and deviations with respect to the design documentation are presented in the technical report. Two monitoring points must be installed on each pier in order to track structural movements during subsequent construction phases and throughout the service life of the bridge. On piers located in water, 360° prisms were installed, whereas benchmarks were installed on piers located on dry land.

The elevations of the benchmarks are determined by geometric levelling using the site 1D geodetic control network. The procedure for determining the displacements of piers located in water will be described in detail in the following section through the applied deformation analysis methodology.

2.5. CONSTRUCTION OF THE BEARING BEAM AND INSTALLATION OF BRIDGE BEARINGS

Geodetic activities during the construction of the bearing beam and the installation of the bridge bearings help ensure the accurate positioning of substructure elements and the proper transfer of loads from the superstructure.

In the initial phase, the designed position of the bearing seat was set out. After the installation of reinforcement and formwork, horizontal and vertical control of the formwork was carried out in accordance with the design documentation. Upon completion of concreting, the positions of the bridge bearings were set out. The controlled parameters in this phase included position, elevation, horizontality, and orientation of the bearing surfaces. Following the installation of the bearings, an as-built geodetic survey was performed and the necessary documentation was prepared for the technical report.

2.6. CONSTRUCTION OF BOX GIRDER SEGMENTS

During the construction of the zero and first segments of the bridge box girder, geodetic works ensure control of the design geometry and accurate positioning of the superstructure elements in the initial stage of construction. The transfer of alignment geometry and elevations provides the basis for compliance with the design documentation and continuity in the construction of subsequent segments.

In the initial phase, geodetic setting out of the scaffolding for the box girder formwork was carried out according to the design position. After the installation of the scaffolding, the position of the formwork was set out, followed by horizontal and vertical control relative to the design specifications. In the subsequent phase, the internal elements of the box girder were set out, including webs, ribs, and internal edges.

Upon completion of the setting out and verification of the scaffolding, formwork, and internal elements, the designed concrete elevations for the zero and first box girder segments were transferred. For each segment, geodetic monitoring points were installed to control deformation. Six points were distributed on the top slab of the box girder, while three points were installed on its bottom slab. The monitoring points were observed in the zero epoch and in the control epoch, i.e., before concreting and after completion of concreting of each segment. This monitoring approach enabled the analysis of structural settlements caused by self-weight and construction-induced loads.

2.7. BALANCED CANTILEVER CONSTRUCTION

As part of the preparation for the cantilever construction of the box girder, the setting out of characteristic details of the form traveler was carried out in accordance with the design documentation (Figure 3). The setting out included reference points and lines necessary for proper assembly and subsequent control of the structural behavior of the form traveler. After completion of the form traveler assembly, a load test was conducted prior to the commencement of concreting. The testing procedure was performed by simulating the designed and increased loads, while geodetic observations were carried out during multiple stages of loading and unloading of the bridge structure. Based on the analysis of the obtained results, it was possible to distinguish between elastic and plastic deformations of the form traveler.

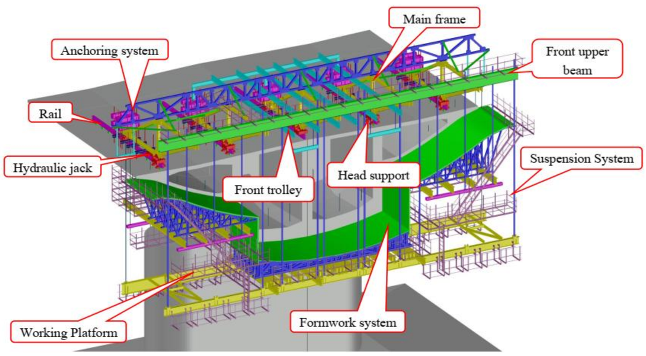

Figure 3. Sketch of form traveler (created by author).

2.8. PYLON CONSTRUCTION

During the construction of the bridge pylon, geodetic activities play a significant role in ensuring the correct geometric positioning of the structure and in controlling pylon movements during the cable tensioning stages. Based on the design documentation, precise setting out of the pylon position on the box girder was carried out.

After the formwork was installed, geodetic control of its horizontal position and elevation was performed relative to the design axes and levels. The pylon was concreted in multiple segments. In segments where cable saddles were planned, the saddles had to be positioned prior to closing the formwork.

During the concreting and erection stages, an as-built survey of the pylon and the installed cable saddles was carried out. Subsequently, a geodetic monitoring point was installed at the top of the pylon to monitor pylon movements during the subsequent construction phases. Observations at the monitoring point were conducted before and after the tensioning of the stay cables, enabling analysis of pylon displacements and deformations induced by the cable system.

2.9. INSTALLATION OF STAY-CABLE GUIDE PIPES

During the construction of the box girder, installing stay-cable guide pipes requires high surveying accuracy. Stay-cable guide pipes directly affect the geometry and proper functioning of the cable system (Figure 4). Geodetic control of the guide pipes positions is carried out in accordance with the data provided by the designer, where the designed camber values are defined for each box-girder segment. The camber values are determined based on the results of geodetic measurements of monitoring points on previously constructed box-girder segments. In this way, a link is established between the actual structural response and the designed geometric parameters of the subsequent segments.

During guide pipes installation, both position and orientation are controlled, and the axis of each guide pipes is verified in space. After installation, an as-built control survey is performed, and both the lower and upper points of the guide pipes axis are measured. The obtained data provide a reference basis for verifying compliance with the design solution and for continuing the installation and tensioning works of the stay cables.

Figure 4. Installation of a stay-cable guide pipes (photography by author).

3. IMPLEMENTATION OF A GEODETIC NETWORK FOR MONITORING BRIDGE PIERS

For the purpose of monitoring the bridge structure piers, a geodetic network has been implemented. Periodic measurements are made from the reference network points to points stabilized on the bridge piers.

The following text describes the mathematical model for processing geodetic measurements and deformation analysis during bridge construction monitoring. The Gauss-Markov model is presented as the basis for the LSA of the free geodetic network and for datum definition. The Karlsruhe deformation analysis method is then described, including a global congruence test for the stability assessment of the reference points and subsequent localization tests for the object points. In the end, the role of JAG3D in implementing the adjustment and statistical testing is described, providing a methodological link to the results presented in the next section.

3.1. GAUSS-MARKOV ADJUSTMENT MODEL

Imperfections of measuring instruments, the operator proficiency, and atmospheric conditions introduce random, systematic, and gross errors into geodetic observation. In order to reduce the influence of these errors, the number of measurements within the network is increased. However, the presence of redundant observations results in an overdetermined system of equations, which requires network adjustment to obtain an optimal solution. Through adjustment, estimates of stochastic variables and the parameters of their distribution are obtained based on the sample of measurements.

For the LSA of geodetic networks, the Gauss-Markov model of adjustment is most commonly applied. This linear mathematical model comprises both a functional and a stochastic component, defining the relationships between stochastic observations and the unknown parameters of the geodetic network, described by the following equations:

$$\mathbf{v} = \mathbf{A} \cdot \hat{\mathbf{x}} + \mathbf{f}, \tag{1}$$

$$K_l = \sigma_0^2 \cdot \mathbf{Q}_l, \tag{2}$$

where $\mathbf{v}$ is the vector of residuals, $\mathbf{A}$ represent the design matrix, $\hat{\mathbf{x}}$ is the vector of unknown parameters, $\mathbf{f}$ is the vector of free terms, K_l is the covariance matrix, $\mathbf{Q}_l$ is the cofactor matrix, and σ_0^2 represent *a priori* variance [19].
The LSA consists of minimising the weighted sum of squared residuals, subject to the condition:

$$\mathbf{v}^{T} \cdot \mathbf{P}_l \cdot \mathbf{v} = min, \tag{3}$$

where $\mathbf{P}_l = \mathbf{Q}_l^{-1}$ is the weight matrix [10]. Accordingly, the least-squares minimization condition results in the normal equations, from which the general solution follows as:

$$\hat{\mathbf{x}} = -\mathbf{N}^{-1} \cdot \mathbf{n} = -\mathbf{Q}_{\hat{x}} \cdot \mathbf{n}, \tag{4}$$

where $\mathbf{N} = \mathbf{A}^T\mathbf{PA}$ is the normal-equation coefficient matrix, $\mathbf{n} = \mathbf{A}^T\mathbf{Pf}$ is the vector of free-term coefficients and $\hat{\mathbf{x}}$ is the vector of estimates of the unknown parameters [10].
In geodetic networks, field measurements determine only the relative positions of points. To obtain a unique coordinate solution, it is necessary to resolve the datum defect of the geodetic network, which causes the singularity of the matrix **N**. The datum of a geodetic network represents a set of parameters that define the position of the network within a coordinate system. These parameters are used to define the network in terms of shape, position, and scale. In free geodetic networks, two approaches are distinguished: the classical datum definition and minimum-trace datum definition [20], [21].

3.2. KARLSRUHE METHOD

The traditional task of deformation analysis refers to the investigation of displacements of critical engineering structures (bridges, dams, high-rise buildings), as well as the surface layer of the Earth's crust, in time and space [19]. For the purposes of deformation analysis in this study, the Karlsruhe method was applied.
The Karlsruhe method is based on the independent adjustment of the zero and control epochs, as well as on their combined adjustment. In the first phase, the observations in each epoch are independently adjusted by the LSA. In the second phase, a combined adjustment of the observed quantities from the zero and control epochs is performed. The combined adjustment of two epochs is carried out under the assumption that the reference points are congruent in both epochs and that the network scale is identical in both epochs [22].
In the first phase, the observed quantities in each epoch are independently adjusted using the method of indirect adjustment:

$$\mathbf{v}_i = \mathbf{A}\mathbf{x}_i + \mathbf{f}_i,\ i = 1,2,\ldots,k, \tag{5}$$

where k denotes the number of epochs [16]. From each individual adjustment, the quadratic form Ω_i is determined, and the total quadratic form for all epochs is obtained by summing the quadratic forms from the adjustments of the individual epochs:

$$\Omega_0 = \sum_{i=1}^{k} \Omega_i = \sum_{i=1}^{k} (\mathbf{v}^T\mathbf{P}_l\mathbf{v})_i = \mathbf{v}^T\mathbf{P}_l\mathbf{v}\,. \tag{6}$$

The total number of degrees of freedom b is obtained by summing the degrees of freedom b_i from the adjustments of the individual epochs. Within the second phase, a combined adjustment of the observed quantities from the zero and control epochs is performed. In the combined adjustment of two epochs, the vector of unknown coordinates is divided into three subvectors:

$$\mathbf{x}^T = (\mathbf{z}^T, \mathbf{x}_1^T, \mathbf{x}_2^T), \tag{7}$$

where $\mathbf{z}$ is the subvector of reference points assumed to be stable, and $\mathbf{x}_1^T$, $\mathbf{x}_2^T$ are the subvectors of points assumed to be unstable. From the combined adjustment, the quadratic form Ω_z is determined, containing information on measurement errors and on the displacements of unstable points. From the quadratic form of the combined adjustment Ω_z, the quadratic form Ω_0, which contains only information on measurement errors, must be subtracted:

$$\Omega_h = \Omega_z - \Omega_0 = \mathbf{v}_z^T\mathbf{P}_l\mathbf{v}_z - \mathbf{v}^T\mathbf{P}_l\mathbf{v}, \tag{8}$$

where the new quadratic form Ω_h contains only information on the displacements of unstable points.
The test statistic is defined as:

$$T = \frac{\Omega_h/f}{\Omega_0/b} \sim F_{1-\alpha,f,b} \tag{9}$$

where $f = (k-1)np_0 - d$, k is number of epochs, n dimension of geodetic network, p_0 number of conditionally stable points and d rank defect of the design matrix **A** [22]. The decision on stability is made by comparing the computed test statistic T with the critical value of the Fisher distribution. The parameters adopted in the Fisher distribution include the significance level $\alpha = 0.05$, while (1-α) denotes quantile of the F-distribution with f and b degrees of freedom.

If $T \leq F_{1-\alpha,f,b}$ then the set of conditionally stable points contains no unstable points. If $T > F_{1-\alpha,f,b}$, then the set of conditionally stable points contains unstable points. It is necessary to determine which points are unstable. For this purpose, the combined adjustments are repeated, successively excluding one conditionally stable point at a time. The adjustment yielding the minimum value of the quadratic form Ω_z indicates that the point excluded in that adjustment should be considered unstable. This point is definitely removed from the set of conditionally stable points, and the entire procedure is repeated without it. The procedure is iteratively repeated until the condition $T \leq F_{1-\alpha,f,b}$ is satisfied. The points that remain in the set of conditionally stable points after fulfilling this condition are declared stable points [9].

3.2.1. Localization of Deformations

The localization of deformations is performed for each point. Under the null hypothesis, it is assumed that point T_j has not been displaced, whereas under the alternative hypothesis it is assumed that point T_j has been displaced. The null hypothesis and the alternative hypothesis are formulated [22]. The test statistic is defined as:

$$T_i = \frac{\hat{\mathbf{d}}_i^T \boldsymbol{Q}_{\hat{\mathbf{d}}_i}^{-1} \hat{\boldsymbol{d}}_i}{h_i \hat{\sigma}_0^2} \sim F_{1-\alpha,h_i,b} \tag{10}$$

where $\hat{\mathbf{d}}_i = \mathbf{B}^T \hat{\mathbf{x}}$ is the displacement vector, $\mathbf{Q}_{\hat{\mathbf{d}}_i} = \boldsymbol{B}^T \boldsymbol{Q}_{\hat{\mathbf{x}}} \boldsymbol{B}$ cofactor displacement matrix, $\mathbf{B}^T = [0 \ldots - \mathbf{I}_i \ldots \mathbf{I}_i \ldots 0]$ where $\mathbf{I}_i$ is the identity matrix, $\hat{\mathbf{x}}$ is adjusted coordinates vector from the joint adjustment of both epochs, $\hat{\sigma}_0^2$ the *a posteriori* unified dispersion factor and h_i = rank($\mathbf{Q}_{\hat{\mathbf{d}}_i}$). If $T_i \leq F_{1-\alpha,h_i,b}$, the null hypothesis is not rejected, i.e., the point is considered stable.

3.3. JAG3D

JAG3D is an open-source LSA software package designed for geodetic and metrological applications. In JAG3D, hybrid terrestrial observations (e.g., levelling, directions, distances, vertical angles) and GNSS baselines can be rigorously combined within a single adjustment. The LSA is formulated in the Gauss-Markov model, and multiple network configurations are supported [17].

For deformation monitoring, JAG3D provides a statistically based congruence analysis between at least two epochs. According to the documentation, single-epoch evaluations are first performed, after which the observations from both epochs are combined and jointly adjusted, allowing correlations to be considered at the observation level. In this framework, reference points are explicitly tested for invariance, and object-point displacement vectors are derived and assessed for statistical significance using F-distributed test statistics [17]. In this paper, JAG3D was employed for the LSA of measurement epochs, the assessment of geodetic reference network stability, and the deformation analysis.

4. RESULTS AND DISCUSSION

For the purpose of assessing the stability of the bridge piers over the Danube, measurements were performed in two epochs. The measurements were carried out during the construction phase in order to determine whether deformations of the bridge piers had occurred due to structural and external influences. The geodetic network was defined by four points serving as the reference framework, as well as four object points located at the tops of the bridge piers. The measurements were performed using a high-precision total station, the Leica TS60, in four gyruses per epoch. The standard deviation of the measured horizontal direction with the Leica TS60 total station is 0.5″, while the distance measurement accuracy is 0.6 mm + 1 ppm.

The bridge geodetic monitoring program includes multiple measurement epochs during construction, while this paper presents an analysis of deformations over a selected interval of two epochs. All results presented in this section are expressed in the adopted construction-project coordinate system of the bridge over the Danube on the bypass road around Novi Sad. The coordinate system was established through a local realization of the national coordinate reference system. After the adjustment of the geodetic control network for the zero and first epochs, a preliminary stability assessment of the control network was carried out. First, a unimodal transformation from the control epoch to the reference epoch was performed. Subsequently, coordinate differences between the transformed coordinates and those of the zero series were determined. Based on the computed coordinate differences, the stability of the network was evaluated using the combined standard deviation. After performing the statistical significance test, it was determined that point JM3 exceeded the critical value and was therefore considered unstable (Table 1).

Table 1. Results of the preliminary stability assessment of the network

Points	ty	tx	tz
JM6	0.17	-0.30	3.33
P10N	0.09	-4.18	-0.56
MAL6	-2.49	-1.32	1.88
JM3	2.17	7.27	-1.13

4.1. DEFORMATION ANALYSIS OF POINTS ON BRIDGE PIERS

Following the successful verification of the stability of the geodetic network, deformation analysis was performed for the points installed on the bridge piers. The objective of the analysis was to determine potential structural displacements between the two observation epochs. The deformation analysis was performed using JAG3D.

Table 2 presents the displacement components $\boldsymbol{\delta}_y, \boldsymbol{\delta}_{x,}$ and $\boldsymbol{\delta}_z$ for the monitored points on piers between a reference and a control epoch, together with the corresponding standard deviations $\sigma_y, \sigma_{x,}$ and σ_z. The point displacements are treated as displacement vectors and evaluated together with their associated dispersion information. The largest displacement was observed at point MP4.2, whereas the smallest displacement was observed at point MP3.1. Table 2 additionally provide the elements of the relative confidence ellipsoids for the displacement vectors of the points on the bridge piers. The semi-axes A, B, and C describe the size of the confidence region formed based on the cofactor information of the estimated displacement vector and the adopted significance level α = 0,05 and test power β = 0,80. The presentation of the values of the confidence semi-axes and their orientation enables the interpretation of displacements with respect to the direction of the major semi-axis, which represents the most unfavorable direction.

Table 2. Accuracy Elements of Points on the Bridge Piers

Points	δ_y[mm]	δ_x[mm]	δ_z[mm]	σ_y[mm]	σ_x[mm]	σ_z[mm]	A [mm]	B [mm]	C [mm]
MP3.1	0.2	3.7	-0.5	1.9	1.7	1.1	1.7	1.9	1.1
MP3.2	2.5	4.7	-1.1	1.7	1.4	1.0	1.4	1.7	1.0
MP4.1	1.7	6.3	-2.9	1.6	1.6	1.0	1.6	1.6	1.0
MP4.2	3.7	8.0	-3.1	1.6	1.4	0.9	1.4	1.6	0.9

Minimal detectable model disturbance (MDMD) represents the threshold value of a geodetic network for a defined significance level α and test power $1-\beta$. The presented results show thresholds of approximately 0.4-3.2 mm in the x direction, 5.4-7.5 mm in the y direction, and 1.0-2.7 mm in the z direction, indicating that the network is least sensitive in the y component (Table 3).

Table 3. Minimal detectable model disturbance

Points	∇_x[mm]	∇_y[mm]	∇_z[mm]
MP3.1	0.4	7.5	1.0
MP3.2	3.2	6.1	1.5
MP4.1	1.6	5.8	2.7
MP4.2	2.5	5.4	2.1

Table 4 presents the results of the local stability test of the points following the deformation analysis performed in JAG3D. For each point, the *a priori* and *a posteriori* test statistics are provided, formed on the basis of the displacement vector and its corresponding cofactor matrix.

For each test statistic, the null hypothesis H_0, assuming the absence of displacement, is tested. The decision is defined as in the column $T \leq q|H_0$, where q denotes the critical value of the corresponding F-distribution. The value "TRUE" indicates that H_0 is not rejected (the point is considered statistically stable), whereas "FALSE" indicates rejection of H_0 (the point is considered statistically unstable) at the adopted significance level α. The deformation analysis of pier points indicated that the largest displacement components were obtained at pier location MP4 (points MP4.1 and MP4.2), while the smallest displacement was obtained at MP3.1. The local stability test shows that the displacements of points MP4.1 and MP4.2 are statistically significant at the adopted significance level, whereas MP3.1 and MP3.2 remain statistically stable. The detected displacements are interpreted as statistically significant construction-stage movements that require continued monitoring, but do not indicate structural instability.

Table 4. Test statistics

Points	T_{prio}	T_{post}	$T \leq q\|H_0$
MP3.1	1.53	1.73	TRUE
MP3.2	3.70	4.18	TRUE
MP4.1	7.44	8.41	FALSE
MP4.2	13.85	15.66	FALSE

Since the measurements were performed during construction, the detected displacements may be associated with construction-stage effects, including changes in structural loading, settlement of structural elements, and environmental influences. Temperature effects may also contribute to the displacement components, particularly in bridge structures. Therefore, the results should be interpreted primarily as geometrically and statistically detected displacements, while the causal interpretation requires additional structural, thermal, and geotechnical information. Figure 5 presents a graphical representation of the network configuration used in the deformation analysis performed in JAG3D. The positions of the reference network points, the object points, and the realized observations are shown. Point JM3 was not adopted as a datum point, as it was identified as unstable during the preliminary stability assessment; consequently, the datum was subsequently realized using the remaining stable points.

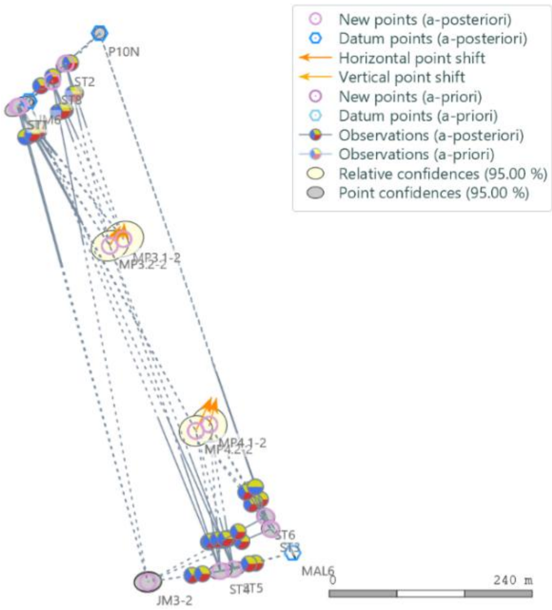

Figure 5. Graphical representation of deformation analysis results in JAG3D (displacement vectors and 95% confidence regions)

The focus of the illustration is on the points located on the bridge piers, for which displacement vectors between the reference and control epochs are displayed. The orange arrows represent the horizontal components of displacement, while the yellow arrows indicate the vertical components. Around the position of each point in the control epoch, relative confidence ellipses are plotted, graphically linking the estimated displacement with its associated dispersion information.

5. CONCLUSION

The paper presents the phases of bridge construction and geodetic monitoring during characteristic construction stages. In the experimental part, the deformation measurements of the points on the bridge piers are presented. Geodetic works performed during the construction of the bridge over the Danube were accompanied by two-epoch deformation monitoring at four points established on the bridge piers, with the aim of detecting potential spatial displacements of the structure. The adjustment and deformation analysis were carried out in the JAG3D environment using a free-network concept and a Karlsruhe approach, enabling a statistically controlled interpretation of epoch-to-epoch differences rather than a purely geometric comparison.
The results showed that MP3.1 and MP3.2 were statistically stable, while MP4.1 and MP4.2 were classified as statistically unstable at the adopted significance level. However, these results should be interpreted in the context of construction-stage monitoring. Statistical significance indicates that the detected displacement is relevant within the adopted mathematical model, but it does not imply structural instability. Based on the reported MDMD, the achieved network sensitivity in the vertical component meets the requirement for reliable detection of the 5 mm settlement defined in the design documentation. The bridge monitoring program includes additional epochs of measurement to determine whether the detected displacements are decreasing or increasing over time. If an increasing trend in displacement is detected in subsequent epochs, the designer and the responsible civil engineers should be informed so that additional control measures can be considered. Overall, the presented workflow demonstrates that JAG3D can be effectively applied to bridge-construction monitoring for reference stability control and statistically supported detection of pier displacements.

ACKNOWLEDGEMENT
The research has been conducted within the project "Enhancement of teaching and scientific research capacities through the application of modern technologies in the fields of civil engineering, geodesy, and disaster risk management and fire safety", developed at the Department of Civil Engineering and Geodesy, Faculty of Technical Sciences, University of Novi Sad, Serbia.

LITERATURE

[1] S. Ašanin, *Engineering geodesy 1*. Belgrade: Ageo, 2003.
[2] C. W.F., *Concepts of Network and Deformation Analysis*. UNSW, 1987.
[3] T. Ninkov, *Designing geodetic networks in engineering geodesy*. Novi Sad: Faculty of Technical Sciences, 2013.
[4] K. Firek and A. Kulpa, “The issues on site surveying during the construction of a bridge span with the cantilever method,” Geomatics and Environmental Engineering, vol. 9, no. 3, pp. 25–34, 2015, doi: 10.7494/geom.2015.9.3.25.
[5] W. Niemeier, “Geodetic techniques for the navigation, guidance and control of construction processes,” in Proc. 3rd IAG Symposium on Geodesy for Geotechnical and Structural Engineering and 12th FIG Symposium on Deformation Measurements, Baden, Austria, May 22–24, 2006.
[6] E. Nowak, R. Malarski, W. Prószyński, A. Sadowska, M. Woźniak, and J. Zaczek-Peplinska, “Research and development work carried out by the Chair of Engineering Geodesy and Measurement and Control Systems, Faculty of Geodesy and Cartography WUT – Thematic scope and achievements,” Reports on Geodesy and Geoinformatics, vol. 100, no. 1, pp. 201–219, 2016, doi: 10.1515/rgg-2016-0014.
[7] T. Mill, A. Ellmann, M. Kiisa, J. Idnurm, S. Idnurm, M. Horemuz, and A. Aavik, “Geodetic monitoring of bridge deformations occurring during static load testing,” The Baltic Journal of Road and Bridge Engineering, vol. 10, no. 1, pp. 17–27, 2015, doi: 10.3846/bjrbe.2015.03.
[8] Z. Sušić, M. Batilović, R. Đurović, M. Marković, and M. Vujinović, “Deformation analysis of Ratkov Laz bridge using Pelzer and IWST method,” *Geod. Vestn.*, vol. 67, no. 04, pp. 487–504, 2023, doi: 10.15292/geodetski-vestnik.2023.04.487-504.
[9] M. Krunislav and A. Ivan R., *Deformation analysis of geodetic networks*. Belgrade: Faculty of Civil Engineering, 1994.
[10] Z. Sušić, M. Batilović, T. Ninkov, I. Aleksić, and V. Bulatović, “Identification of movements using different geodetic methods of deformation analysis,” *Geod. Vestn.*, vol. 59, no. 03, pp. 537–553, 2015, doi: 10.15292/geodetski-vestnik.2015.03.537-553.
[11] O. Heunecke and W. Welsch, “Terminology and Classification of Deformation Models in Engineering Surveys”.

[12] W. Walter M. and H. Otto, "Models and terminology for the analysis of geodetic monitoring observations," *10th FIG Int. Symp. Deform. Meas.*.

[13] B. Konakoglu, "Deformation Analysis Using Static, Kinematic and Dynamic Geodetic Deformation Models with GNSS: Deriner Dam, Artvin, Turkey," *Exp. Tech.*, vol. 45, no. 5, pp. 645–660, Oct. 2021, doi: 10.1007/s40799-020-00435-z.

[14] M. Yalcinkaya and T. Bayrak, "Comparison of Static, Kinematic and Dynamic Geodetic Deformation Models for Kutlugun Landslide in Northeastern Turkey," *Nat. Hazards*, vol. 34, no. 1, pp. 91–110, Jan. 2005, doi: 10.1007/s11069-004-1967-2.

[15] V. Hamza, T. Ambrožič, and B. Stopar, "Deformation analysis: the Caspary approach," *Geod. Vestn.*, vol. 64, no. 01, pp. 68–88, 2020, doi: 10.15292/geodetski-vestnik.2020.01.68-88.

[16] D. E. Wells and K. Edward J, *The method of least squares*, vol. 18. Canada: Department of Surveying Engineering, University of New Brunswick, 1971.

[17] "Java Applied Geodesy 3D," JAG3D. Available: https://software.applied-geodesy.org/en/.

[18] *ISO 17123, Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*.

[19] Z. Susic, "A geodynamical analysis of Earth's crust movements of regional character," PhD thesis, University of Novi Sad, Novi Sad, 2013. doi: 10.2298/NS20131224SUSIC.

[20] M. Batilović, "Robust estimation of deformations in free geodetic networks usingevolutionary optimisation algorithms," PhD thesis, University of Novi Sad, Novi Sad, 2021. [Online]. Available: https://nardudejan s.mpn.gov.rs/handle/123456789/18117.

[21] H. B. Papo, "Datum Accuracy and Its Dependence on Network Geometry," in *Geodesy-The Challenge of the 3rd Millennium*, E. W. Grafarend, F. W. Krumm, and V. S. Schwarze, Eds., Berlin, Heidelberg: Springer Berlin Heidelberg, 2003, pp. 379–386. doi: 10.1007/978-3-662-05296-9_39.

[22] Z. Susic, "Geometric deformation analysis in free geodetic networks: case study for Fruska Gora in Serbia," *Acta Geodyn. Geomater.*, pp. 341–355, Aug. 2017, doi: 10.13168/AGG.2017.0017.