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Miljana Todorović Drakul, University of Belgrade, mtodorovic@grf.bg.ac.rs
Sanja Grekulović, University of Belgrade, sanjag@grf.bg.ac.rs
Dušan Petković, University of Belgrade, dpetkovic@grf.bg.ac.rs
Oleg Odalović, University of Belgrade, odalovic@grf.bg.ac.rs

TESTING METHODS WITH DIFFERENT DEGREES OF SAMPLING IN DETERMINING SYSTEMATIC INFLUENCES AND MEASUREMENT UNCERTAINTIES OF LINEAR MEASURING DEVICES

Abstract

This work tested several procedures to determine the linear measuring device's systematic influences and measurement uncertainty. Three methods were developed that differ in the degree and density of sampling during measurement and were compared with the existing method used for calibrating linear measuring devices in the Metrology Laboratory accredited for calibrating angle and length, Institute of Geodesy and Geoinformatics, Faculty of Civil Engineering, in Belgrade. The goal is to determine the possibility of introducing these methods into the calibration process without violating the accuracy and precision measures and to meet the assumed measurement uncertainty requirements of measuring devices and equipment.

Keywords: measurement uncertainty, systematic influence, precision, linear measuring device

ТЕСТИРАЊЕ МЕТОДА РАЗЛИЧИТОГ СТЕПЕНА УЗОРКОВАЊА КОД ОДРЕЂИВАЊА СИСТЕМАТСКИХ УТИЦАЈА И МЈЕРНЕ НЕСИГУРНОСТИ ЛИНЕАРНИХ МЈЕРИЛА

Сажетак

У овом раду тестирано је неколико поступака за одређивање систематских утицаја и мјерне несигурности линеарних мјерила. Развијене су три методе које се разликују у степену и густини узорковања приликом мјерења и поређене су са постојећом методом која се користи за еталонирање линеарних мјерила у Метролошкој лабораторији акредитованој за еталонирање мјерила угла и дужине, Института за геодезију и геоинформатику, Грађевинског факултета, у Београду. Циљ је утврдити могућност увођења ових метода у поцес еталонирања линеарних мјерила, а да се не наруше мјере тачности, прецизности и да се задовоље претпостављени захтјеви мјерне несигурности мјерила и опреме.

***Кључне ријечи**: мјерна несигурност, систематски утицај, прецизност, линеарна мјерила*

1. INTRODUCTION

Linear measuring devices represent equipment used by many beneficiaries for various needs and, as such, should satisfy the appropriate properties. The linear measuring devices used are ribbons, measuring tapes, measuring rods, levelling rods and rulers. The measuring devices can be materialised from glass, steel or other materials, on which the dimensions are marked with lines of a certain thickness. When checking the measuring devices, sampling and reading the marked division on the devices is done according to a specific procedure and with appropriate equipment. The task of this work is to determine and check the optimal sampling density that would satisfy the assumed quality.

All measurements were made in the Metrology Laboratory accredited for the calibration of angle and length gauges, Institute of Geodesy and Geoinformatics, Faculty of Civil Engineering, in Belgrade, according to the method given in the Working instructions for calibration of leveling bars, rulers, measuring bars and measuring tapes and is in accordance with the standard ISO 17123-1 - Optics and optical instruments - Field procedures for testing geodetic and surveying instruments — Part 1: Theory. The measuring system used for the calibration of rulers, i.e. all linear measures, consists of an HP 5508A laser interferometer and a measuring bench on which the interferometer and mesuring device is placed [1].

In this paper, several methods were tested, which differ in the degree and density of sampling during measurement, for determining the systematic influences and measurement uncertainty of linear measuring devices. A measuring ruler with a length of 500 mm was used for these purposes.

This research can be presented through the following steps:

- Presentation of linear measuring device and measuring systems located in the Metrology Laboratory of the Institute of Geodesy and Geoinformatics of the Faculty of Civil Engineering in Belgrade;
- Elaboration of various methods of precise determination of systematic influences and measurement uncertainty of linear scales and sampling and reading of the marked division on the scale according to a specific procedure and in the appropriate number of repetitions;
- Assessment of measurement parameters and recording of the existence of systematic influences and determination of measurement uncertainty for all projected sampling combinations;
- Testing hypotheses about the equality of dispersions and the equality of expected values;
- Display of the obtained results and determination of the optimal sampling solution according to the given measurement procedure;

2. EQUIPMENT: LINEAR MEASURING DEVICE AND MEASURING SYSTEMS

The linear measuring device used for this experiment is a metal ruler with a length of 500 mm (Figure 1). The division was carried out on both sides in two different proportions. For this test, a division performed on a scale of 1:1000 was used, where dashes indicate millimetres, and numbers show centimetres. Crucial for the choice of this ruler was the material from which it was made (metal with a low degree of deformity) and the high quality of the engraved division.

Figure 1. Measuring device - a metal ruler with a length of 500 mm

The measuring system used for ruler calibration consists of an HP 5508A laser interferometer (Figure 2) and a measuring bench (Figure 3) on which the interferometer and measuring device are placed. The measuring bench is designed to ensure precise positioning and system stability. The interferometer is placed on sliding tracks consisting of prisms that serve for precise positioning and adjustment of the device. The division reads with a digital reader and a microscope (Figure 4).

Figure 2. HP 5508A laser interferometer

Figure 3. Measuring bench

Figure 4. Digital reader - microscope

During the measurement, data on the temperature of the working environment and the temperature of the material of the measuring device are taken using sensors:

- HP 10757B, which measures the temperature of the ruler material,
- And the HP 10751B sensor that measures the air temperature of the working environment (laboratory).

In addition to the above data on temperatures, information on the humidity of the air in the laboratory is taken from the thermohygrometer Mastech (MS6508) in order to monitor the state of the atmospheric conditions in the laboratory [1].

Table 1 shows the basic characteristics of the measuring equipment.

Table 1. Characteristics of measuring equipment

R. no.	Name	Manufacturer	Type	Ser. number	Measuring range	Uncertainty
1	Measuring bench	-	-	-	(0 – 6000)mm	-
2	Laser interferometer	Hewlett Packard	HP 5528A	2532A02 552	Up to 15 m	5*10-7
3	Air temperature sensor	Hewlett Packard	HP 10751B	-	(0 – 40) °C	0,50 °C For the range (15 – 25) °C
4	Air pressure sensor	Hewlett Packard	HP 10751B	-	(517,2 – 775,7) mmHg	1,40 mmHg For the range (15 – 25) °C
5	Material temperature sensor	Hewlett Packard	HP 10757B	-	(0 – 40) °C	0,10 °C

2.2. INSTALLATION OF MEASURING EQUIPMENT AND LINEAR MESURING DEVICE

The laser interferometer is placed on the interferometer plate of the work bench, where it is connected to the power supply, as well as to the sensors HP 10757B and HP 10751B. After placing the prisms of the interferometer, on the fixed and movable support of the workbench, the interferometer is switched on, after which the process of achieving the optimal temperature begins for the correct operation of the interferometer laser (the time period for achieving the optimal temperature is never longer than 5 minutes). After reaching the optimal temperature, the laser and prisms are adjusted in order to establish the interferometer's operation.

Figure 5. Installation of the ruler on the measuring bench

The setting is performed as follows:

The plate on the laser head is turned to block the laser return beam. It is checked whether the red dot on sight (crosshair) moves when the slide is moved from one end to the other end of the rail of the measuring bench. If the red dot moves then alignment is done until the red dot rests on the crosshairs of the laser head when the carriage is moved from one end of the bench rail to the other.

The ruler is placed on the measuring bench, that is, on the supports of the linear measuring device of the measuring bench, parallel to the direction defined by the prism supports located on the rail of the measuring bench (Figure 5). After installation, the ruler is levelled using the workbench's measuring device support ruler. Then, using the binoculars of the measuring bench, the unique focus of the binoculars along the entire length of the ruler additionally ensures the levelling.

3. PROPOSED MEASUREMENT METHODS AND MATHEMATICAL BASIS OF PROCESSING MEASUREMENT RESUTS

The measurement procedure is performed as follows:
The lengths between the zero line (partition) and the other subdivision lines of the scale, including the last subdivision line (partition), are measured. The number of measuring points depends on the length of the measuring scale of the measuring devices, in this case the ruler. The length is measured from the zero division to each meter division. For measuring tapes over 5 meters long, every meter is measured.
To determine the central line of each division of the ruler, the middle of the division (full division line) coincides with the microscope's crosshair. The positioning system is set from one coordinate axis (reticle axis) placed in the division's central line or at the beginning of the division (Figure 6). Five measurements are taken at each measurement point.

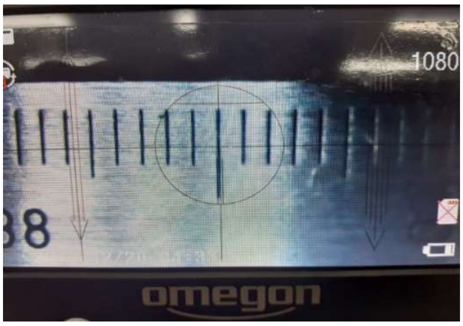


Figure 6. Coincidence of the reticle with the edge of the division

In our case, for testing purposes, we used four sampling methods:
Method 1 - For the first and last decimeter (dm) measure every centimeter (cm) and for the first and last centimeter (cm) measure every millimeter (mm).
Method 2 - Every cm was measured
Method 3 - For the first and last dm, measure every cm
Method 4 – Every odd cm was measured

3.2. PROCESSING OF MEASUREMENT RESULTS

Processing of measurement data implies application of statistical calculations to the results. For each measurement point, the mean value and standard deviation are calculated. The mean value of the measurement at each point is calculated using the expression [2], [3]:

$$\bar{C} = \frac{\sum_{i=1}^{5} C_i}{5} \quad (1)$$

where C_i represents the measurement of one subdivision.
The standard deviation is calculated according to the expression (ISO 17123-1) [2]:

$$\sigma_C = \sqrt{\frac{\sum_{i=1}^{5}(\bar{C}-C_i}{4}} \quad (2)$$

After that, the deviation of the measured value from the nominal value is calculated as:

$$\Delta C_i = \bar{C_j} - C_{j\,nom}, j = 1, n \quad (3)$$

where n depends on the length of the ruler.
A diagram of differences is drawn based on the nominal values of the ruler and the corresponding deviations measured from those nominal values. The nominal values of the scale are applied on the

X-axis of the diagram, and the deviation values on the Y-axis. After drawing the difference diagram on the chart, the equalising right of deviation from the nominal values is illustrated in the form:

$$Y = a * X + b \tag{4}$$

Determination of the unknown parameters of the equation of the line is done by adjustment according to the least squares method. This determines the unknown parameters of lines a and b and their standard deviations. Mathematical statistics methods decide whether the parameter a equals zero in the confidence interval 1-α. Null hypothesis: a = 0 is accepted if the following condition is satisfied:

$$|\mathrm{a}| \leq S_a * t_{1-\frac{\alpha}{2}}(\mathrm{v}) \tag{5}$$

where $t_{1-\alpha/2}$ is the quantile of the student distribution depending on the number of degrees of freedom (v). Otherwise, the null hypothesis is rejected, and the parameter a has the value determined by the adjustment.

3.3. MEASUREMENT UNCERTAINTY

3.3.1. MATHEMATICAL MODEL OF MEASUREMENT

The expression for the mathematical model of measurement can be written in the following form [4]:

$$e = l_m \cdot (l + \alpha_m \cdot \theta_m) - l_{LI} + e_{dif} + e_{cos} + e_{cos2} + e_{dz} + e_a \tag{6}$$

Where:
e - deviation (measurement result) at 20°C,
l_m - the length of the path between the reference position and the measurement position,
α_m - linear temperature expansion coefficient of the instrument,
θ_m - deviation of the instrument temperature from 20°C,
l_{LI} - corrected length indicated by LI,
e_{dif} - error in the difference between the current and initial division mark readings of the instrument,
e_{cos1} - cosine error in measurement due to misalignment of the instrument (expected value is 0), e_{cos2} - cosine error in measurement due to misalignment of the laser beam (expected value is 0),
e_{dz} - dead zone error (expected value is 0),
e_a - error caused by angular deviation of the telescope (expected value is 0).

3.3.2. THE STANDARD UNCERTAINTY OF THE ESTIMATE OF THE INPUT QUANTITY AND THE COMBINED STANDARD UNCERTAINTY OF THE MEASUREMENT

The standard measurement uncertainty according to EA-4/02 is:

$$U_c^2(\mathrm{e}) = c_{lm}^2 \cdot u^2(l_m) + c_{\alpha m}^2 \cdot u^2(\alpha_m) + c_{\theta m}^2 \cdot u^2(\theta_m) + c_{l_{LI}}^2 \cdot u^2(l_{LI}) + c_{e_{raz}}^2 \cdot u^2(e_{raz}) + c_{e_{cosl}}^2 \cdot u^2(e_{cosl}) + c_{e_{cos2}}^2 \cdot u^2(e_{cos2}) + c_{e_{mp}}^2 \cdot u^2(e_{mp}) + c_{e_a}^2 \cdot u^2(e_a) \tag{7}$$

Where c_i are the partial derivatives of the function (7):

$$c_{l_m} = \frac{\partial f}{\partial l_m} = 1 + \alpha_m \cdot \theta_m \approx 1; \theta_{max} = \pm 1°C \tag{8}$$

$$c_{\alpha_m} = \frac{\partial f}{\partial \alpha_m} = \theta_m \cdot l_m \tag{9}$$

$$c_{\theta_m} = \frac{\partial f}{\partial \alpha_m} = \alpha_m \cdot l_m \tag{10}$$

$$c_{l_{LI}} = \frac{\partial f}{\partial l_{LI}} = -1 \tag{11}$$

$$c_{e_{raz}} = \frac{\partial f}{\partial e_{raz}} = 1 \tag{12}$$

$$c_{e_{cosl}} = \frac{\partial f}{\partial e_{cosl}} = 1 \tag{13}$$

$$c_{e_{cos2}} = \frac{\partial f}{\partial e_{cos2}} = 1 \quad (14)$$

$$c_{e_{mp}} = \frac{\partial f}{\partial e_{mp}} = 1 \quad (15)$$

$$c_{e_a} = \frac{\partial f}{\partial e_a} = 1 \quad = 1 \quad (16)$$

The standard uncertainties of the input values are calculated (evaluated) for the applied equipment and method, as well as for assumed measurement conditions.

a) Uncertainty of the path length between the reference and measurement positions $u(l_m)$.

This uncertainty arises due to compensating for the change in the speed of light in external conditions compared to the speed of light in a vacuum. Determination of external ambient conditions is performed using HP 10751 sensors. For a temperature interval that remains constant in the laboratory, ranging from (15 – 20)°C, and according to the manufacturer's specifications, the measurement uncertainty due to the compensation of the speed of light is:

$$u(l_m) = (1.5 * 1)\mu m, \text{ for 1 in meter.} \quad (17)$$

b) Uncertainty of the linear coefficient of thermal expansion $u(\alpha_m)$

As rulers are made of different materials, their thermal expansion coefficients vary. The measuring instruments under examination are made of invar, fibreglass, or steel, each having thermal expansion coefficients in the range of (1 - 12) µm/m°C. The deviation interval of the change in thermal expansion coefficients is $\pm 1 \times 10^{-6}$ °C^{-1}.

The standard uncertainty, assuming a rectangular distribution, is:

$$u(\alpha_m) = \frac{1*10^{-6} \cdot C^{-1}}{\sqrt{3}} = 0.58 * 10^{-6} \cdot C^{-1} \quad (18)$$

c) Uncertainty of the temperature deviation $u(\theta_m)$

The standard uncertainty of temperature measurement for the used HP 10757 sensor is 0.1 °C, i.e.:

$$u(\theta_m) = 0.10 \cdot ℃ \quad (19)$$

d) Uncertainty of the LI indication $u(l_{LI})$

The standard uncertainty for this type of device has a value:

$$u(l_{LI}) = (0.1 * 1)\mu m, \text{ for 1 in meter.} \quad (21)$$

e) Uncertainty due to the error in reading the current and initial division mark of the measuring instrument $u(e_{dif})$

The calibration procedure involves forming the difference in readings between the values at individual positions of the measuring instrument and the initial values (zero, starting point, on tapes and rulers, or the first decimeter on leveling sticks). When reading both values, there is a reading error with a measurement uncertainty $u(e_{cur})$ at individual (current) positions of the measuring instrument and $u(e_{ref})$ at the initial (reference) value. Since the measurements are taken under the same conditions and by the same metrologist, it can be considered that the measurement uncertainties of reading are the same, i.e., $u(e_{cur}) = u(e_{ref}) = u(e_{reading})$. The total measurement uncertainty of the reading difference then amounts to:

$$u(e_{diff}) = u(e_{reading}) * \sqrt{2} \quad (22)$$

The measurement uncertainty of the reading is determined by a statistical estimate from measurements carried out by two individuals, each doing 60 readings individually at the reference point. The measurement uncertainty of the reading is determined separately for levelling sticks and separately for rulers and measuring tapes due to the different qualities of the division application. The standard deviation of these measurements, accepted as the standard uncertainty, is:

$$s = u(e_{reading}) = 0.24\ \mu m \text{ - for leveling sticks and} \quad (23)$$

$$s = u(e_{reading}) = 1.97\ \mu m \text{ - for rulers and measuring tapes.} \quad (24)$$

Accordingly, it follows:

$$s = u(e_{diff}) = 0.34\ \mu m - \text{for leveling sticks} \quad (25)$$

$$s = u(e_{diff}) = 2.79\ \mu m - \text{for rulers and measuring tapes.} \quad (26)$$

f) Uncertainty caused by cosine error $u(e_{\cos})$ (for laser and ruler)

The maximum expected value of the standard uncertainty due to cosine error, according to the manufacturer's specification, is:

$$u(e_{cos}) = (0.03 * 1)\mu m,$$, for 1 in meter. (27)

g) Uncertainty caused by dead zone $u(e_{\mathrm{dz}})$

This component is negligible in the specific case.

h) Uncertainty caused by Abbe error $u(e_{\mathrm{a}})$

This component is caused by different angles of the telescope and the LI reflector along the measurement path. In specific lighting conditions, this error is not pronounced, i.e., its value is considered to be 0.

In tables 2 and 3, values for the standard uncertainties of input quantity estimations for the lower limit of the measurement range 1 mm and 1000 mm for rulers and measuring tapes are presented.

Table 2. Standard uncertainties of input quantity estimations for the lower limit of the measurement range (1 mm) for rulers and measuring tapes.

Quantity	Estimated Quantity	Standard Uncertainty	Distribution	Sensitivity Coefficient	Uncertainty Contribution
l_m	1 mm	0.0015 µm	normal	1	0.00150 µm
α_m	11°C	0.58×10^{-6} °C^{-1}	rectangular	0.05 °C^{-1}	0.00032 µm
θ_m	0°C	0.1°C	normal	0.58×10^{-6} °C^{-1}	0.00006 µm
l_{LI}	1 mm	0.0001 µm	normal	1	0.00100 µm
e_{diff}	1 mm	2.79 µm	normal	1	2.79000 µm
e_{cos1}	0.03*10^{-6} µm	0.00003 µm	normal	1	0.00003 µm
e_{cos2}	-	-	-	-	-
e_a	0	0.0 µm	uniform	1	0.00000 µm
				Total:	2.79291 µm

Table 3. Standard uncertainties of input quantity estimations for the upper limit of the measurement range (1000 mm) for rulers and measuring tapes.

Quantity	Estimated Quantity	Standard Uncertainty	Distribution	Sensitivity Coefficient	Uncertainty Contribution
l_m	1000 mm	1.5 µm	normal	1	1.500 µm
α_m	11°C	0.58×10^{-6} °C^{-1}	rectangular	0.05 °C^{-1}	0.317 µm
θ_m	0°C	0.1°C	normal	0.58×10^{-6} °C^{-1}	0.058 µm
l_{LI}	1000 mm	0. 10 µm	normal	1	1.000 µm
e_{diff}	1000 mm	2.79 µm	normal	1	2.790 µm
e_{cos1}	0.03*10^{-6} µm	0.03 µm	normal	1	0.030 µm
e_{cos2}	-	-	-	-	-
e_a	0	0.0 µm	uniform	1	0.000 µm
				Total:	5.695 µm

According to all the above, the **combined standard uncertainty** estimation of input quantities under the best possible measurement conditions can be expressed by the equation (calculated from tables 2-3):

$$u = 2.79\ \mu m + 5.695 * 10^{-6} * l_{\mu m}$$ – for rulers and measuring tapes. (28)

In accordance with EA-4/02, the expansion factor k=2 is used for calculating the **expanded uncertainty**:

$$u = 5.58\ \mu m + 11.39 * 10^{-6} * l_{\mu m}$$ – for rulers and measuring tapes. (29)

4. MEASUREMENT RESULTS, ASSESSMENT OF SYSTEMATIC INFLUENCES AND MEASUREMENT UNCERTAINTIES OF THE LINEAR MEASURING DEVICES

As previously described, a Laser Interferometer was used for testing methods to determine systematic influences and measurement uncertainties of linear measuring devices.

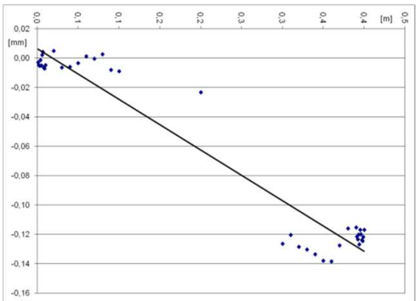
After positioning the interferometer prism on the fixed and movable supports of the workbench, turning on the interferometer, and achieving the optimal temperature for the correct operation of the laser interferometer, measurements or sampling were performed on the ruler described in Chapter 2, Figure 1.

The lengths between the zero line (subdivision) and other division lines of the measuring devices were measured, including the last division line (subdivision). To determine the central line of each subdivision of the measuring devices, the coincidence of the midpoint of the subdivision (full division line) with the crosshair of the telescope's reticle was achieved. The positioning system is set from a single coordinate axis (reticle axis), which is aligned with the central line of the subdivision or the beginning of the subdivision. Five measurements were performed at each measurement point.

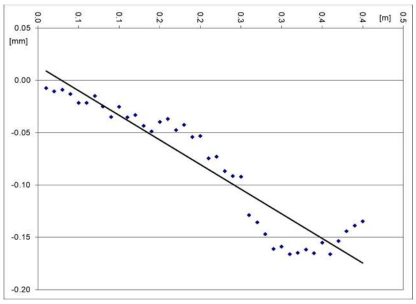
In our case, for the testing purposes, we used four sampling methods:

- Method 1 - For the first and last decimeter (dm), every centimeter (cm) was measured, and for the first and last centimeter (cm), every millimeter (mm) was measured. A total of 40 samplings were performed with five coincidences each, resulting in a total of 200 measurements.
- Method 2 - Every centimeter (cm) was measured. A total of 40 samplings were performed with five coincidences each, resulting in a total of 200 measurements.
- Method 3 - For the first and last decimeter (dm), every centimeter (cm) was measured. A total of 22 samplings were performed with five coincidences each, resulting in a total of 110 measurements.
- Method 4 - Every odd centimeter (cm) was measured. A total of 20 samplings were performed with five coincidences each, resulting in a total of 100 measurements.

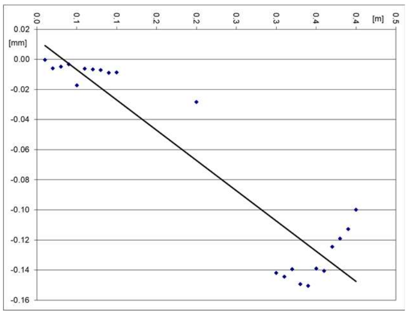
The following measurement graphs were obtained based on the measured values (Figure 7). The graphs show different degrees and densities of levelling sampling for all four applied methods.



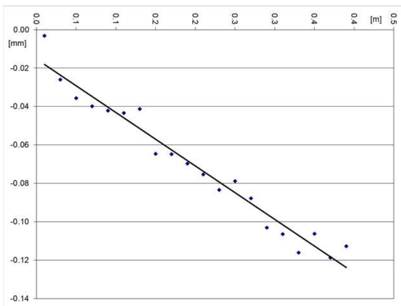
Method 1



Method 2



Method 3



Method 4

Figure 7. The degree of sampling of the ruler

The measurement results were processed using the least squares adjustment method. The parameters for the addition constant, multiplication, and expanded measurement uncertainty for measurements with the interferometer have been calculated.

These results are presented in Table 4 for all four methods, and Table 5 represents their differences. Table 6 contains standard deviations and measurement deviations from the mean value.

Table 4. Parameters for the addition constant, multiplication, and expanded measurement uncertainty for measurements with the interferometer.

Method	Addition constant [mm]	Multiplication	Expanded measurement uncertainty of the reading u(r) [mm]	Expanded measurement uncertainty of other influences u(odher) [mm]
1	0.0063	0.99965528	0.008	0.000029
2	0.0134	0.99952976	0.013	0.000054
3	0.0134	0.99959825	0.018	0.000068
4	-0.0152	0.99972140	0.006	0.000027

Table 5. Differences in the obtained results.

Method	Differences of additions constants [mm]	Differences of multiplication constants	Differences of readings u(r) [mm]	Differences in other influences u(odher) [mm]
1-2	*0.007*	*0.00012552*	*0.005*	*0.000025*
1-3	*0.007*	*0.00005703*	*0.010*	*0.000039*
1-4	*0.022*	*0.00006612*	*0.002*	*0.000002*
2-3	*0.000*	*0.00006849*	*0.005*	*0.000014*
2-4	*0.029*	*0.00019164*	*0.007*	*0.000027*
3-4	*0.029*	*0.00012315*	*0.012*	*0.000041*

Table 6. Values of standard deviation.

Method	Mean value of standard deviation [mm]	The mean value of the deviation [mm]
1	0.006	-0.064
2	0.008	-0.083
3	0.006	-0.071
4	0.006	-0.071

Based on the calculated results, according to Table 2, the following conclusions can be drawn:

- The highest addition and multiplication constants values are obtained using Method 4 and are -0.0152 mm and 0.99972140, respectively.
- The smallest value of the addition constant is obtained using Method 1 and is 0.063 mm, while the smallest value of the multiplication constant is obtained using Method 2 and is 0.99952976.
- The total expanded measurement uncertainty of interferometer measurement (influence of reading uncertainty and other influences) is the smallest when using Method 4, and the largest when using Method 3.

Regarding the differences in the results obtained using different methods, and according to Table 3, we can conclude:

- The largest difference in addition constants is between Method 2 and 3 and Method 4, amounting to 0.029 mm, while the smallest is between Method 2 and Method 3, and it is 0 mm.
- Between Method 2 and Method 4 is the largest difference of multiplication constants (0.00019164), and the smallest between Methods 1 and 3 (0.00005703).

- The largest difference in the total expanded measurement uncertainty (for both parameters) is between Methods 3 and 4 (0.012 mm and 0.000041 mm), while the smallest is between Methods 1 and 4 (0.002 mm and 0.000002 mm).

Table 3 shows that:

- the mean value of the standard deviation is the highest with Method 3, while it is the same with the other methods i
- the mean value of the deviation is the highest with Method 2, while it is the lowest with Method 1.

As seen previously, maximum differences in additional constants occur at the hundredth part of a millimetre, specifically when differences involving measurement data according to Method 4 are considered. This is somewhat justified, considering that the fourth method has the smallest number of measurements. Differences between other methods are in the order of thousandths of a millimetre. A similar situation applies to differences in multiplication constants.

However, to make a final judgment about the quality of the proposed methods, it is necessary to perform testing of dispersions and expected measurement values.

4.2. TESTING HYPOTHESES ABOUT THE EQUALITY OF DISPERSIONS AND THE EQUALITY OF EXPECTED VALUES

We will test the equality of dispersions to verify whether the application of the new methods yields measurement results with the same accuracy as the results obtained by Method 1, which is used as the fundamental method in such examinations. We will use the F-test to test the hypotheses:

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2 \tag{30}$$

with test statistics [5]:

$$F = \frac{m_A^2}{m_B^2} \tag{31}$$

where are they:

m_A^2 = max(m_1^2, m_2^2) and m_B^2 = min(m_1^2, m_2^2) evaluation of dispersions σ_1 and σ_2 and $F = |H_0 F(f_B, f_A)$, with degrees of freedom f_A and f_B, so the decision of the test is:

$$F < g \text{ - accepted } H_0 \tag{32}$$

where is

$$g = F_{1-\alpha}(f_B, f_A) \tag{33}$$

with significance level α.

The results of the testing are presented in tables 7, 8 and 9.

Table 7. Values of the test statistics for comparing the equality of dispersions for Method 1 and Method 2.

	Method 1	Method 2	Test decision
n	40	40	F<g H_0 is accepted - EQUAL DISPERSIONS
σ^2_0	0,000033486	0,000057738	
σ_0	0,006	0,008	
f	39	39	
F	1,724		
$g_{0,95}$(39,39)	1,704		

Table 8. Values of the test statistics for comparing the equality of dispersions for Method 1 and Method 3.

	Method 1	Method 3	Test decision
n	40	22	F<g H_0 is accepted - EQUAL DISPERSIONS
σ^2_0	0,00003349	0,00003395	
σ_0	0,006	0,006	
f	39	21	
F	1,014		
$g_{0,95}(21,39)$	1,833		

Table 9. Values of the test statistics for comparing the equality of dispersions for Method 1 and Method 3.

	Method 1	Method 3	Test decision
n	40	20	F<g H_0 is accepted - EQUAL DISPERSIONS
σ^2_0	0,000033486	0,000031250	
σ_0	0,006	0,006	
f	39	19	
F	1,072		
$g_{0,95}(39,19)$	2,029		

Considering that equal dispersions are obtained for all three comparisons, we can assume that the results obtained by using Method 2, 3, and 4 have the same accuracy as the results obtained by Method 1. This means that the density, i.e., the degree of sampling in these methods, does not compromise accuracy.

As dispersions from the previous testing are equal, and results for additive and multiplicative constants are obtained from the adjustment, we will perform a test of equality of their expected values, assuming that observations belong to normally distributed sets, according to the expression [5]:

$$Z = \frac{d}{\sigma_d} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \approx N[0,1] \tag{34}$$

where σ_1 and σ_2 are known dispersions and $\overline{X}_1$ i $\overline{X}_2$ are parameter estimate values a_1 and a_2. Parameters a_1 and a_2 represent estimated additive or multiplicative constant values from two compared methods. Like testing dispersions, we will take Method 1 as the reference and test the other three.

The test hypothesis is:

$$H_0: a_1 = a_2$$
$$H_a: a_1 \neq a_2 \tag{35}$$

The test decision is then:

$$|Z| \geq q \tag{36}$$

and reject H_0 and a_1 is not equal to a_2, where q is:

$$q = Z_{1-\frac{\alpha}{2}} \tag{37}$$

Based on the previous, we obtained results presented in Tables 10 and 11.

Table 10. Results of the test of equality of expected values of the additive constant

Additive constant	Method 1/ Method 2	Method 1/ Method 3	Method 1/ Method 4
Z	-1,798	-1,271	7,440
$q_{0,99}$	2,576	2,576	2,576
Test decision	Z < q – H_0 is accepted and a_1=a_2 is valid	Z < q – H_0 is accepted and a_1=a_2 is valid	Z > q - we reject H_0 and a_1=a_2 does not hold

Table 11. Results of the test of equality of expected values of the multiplication constant

Additive constant	Method 1/ Method 2	Method 1/ Method 3	Method 1/ Method 4
Z	0,032	0,010	-0,023
$q_{0,99}$	2,576	2,576	2,576
Test decision	Z < q – H_0 is accepted and a_1=a_2 is valid	Z < q – H_0 is accepted and a_1=a_2 is valid	Z < q – H_0 is accepted and a_1=a_2 is valid

According to the values obtained from Tables 10 and 11, the following conclusions can be drawn: there is no significant difference in measurement results using these methods, except in the case of determining the addition constant using Method 4. For all other cases, it is considered that the values of the addition and multiplication constants are obtained with satisfactory accuracy.

5. CONCLUSION

In this study, several procedures for determining systematic influences and measurement uncertainties of linear measuring instruments were tested. Three methods were developed, differing in the degree and density of sampling during measurements, and they were compared with the existing method used for calibrating linear measuring devices in the Metrology Laboratory accredited for angle and length calibration at the Institute of Geodesy and Geoinformatics, Faculty of Civil Engineering, in Belgrade.

After conducting measurements, sampling, data processing, and performing appropriate statistical analyses, it can be concluded that Method 1, which also represents the standard procedure for testing the precision of linear measuring instruments, provides the best results. However, other methods do not lag behind in terms of accuracy. Method 4 yields poorer results in determining the addition constant, while Method 2 and Method 3 can be reliably used as alternatives to Method 1. For all other cases, it is considered that the values of the addition and multiplication constants are obtained with satisfactory accuracy.

Considering the time and number of measurements expended, and based on the obtained results, Method 3 can be considered the optimal solution for such tasks.

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