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Review paper
Doi 10.61892/STP202601033M
ISSN 2566-4484

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A REVIEW OF COMPUTATIONAL CONTACT MECHANICS USING ISOGEOMETRIC ANALYSIS

Abstract:

Contact mechanics plays a central role in many engineering applications and remains a challenging topic in numerical simulation due to the presence of strong nonlinearities. This paper presents a review of contact formulations developed within the framework of isogeometric analysis (IGA). After outlining classical finite element contact approaches and their limitations, the fundamental concepts of B-splines and NURBS are introduced, followed by a brief overview of the application of IGA in structural mechanics. Recent advances in isogeometric contact formulations are then reviewed, demonstrating that IGA-based contact methods can achieve improved accuracy, robustness, and computational efficiency compared to conventional finite element techniques.

Keywords: isogeometric analysis, NURBS, contact mechanics, FEM, contact algorithms

ПРЕГЛЕД ПРИМЈЕНЕ ИЗОГЕОМЕТРИЈСКЕ АНАЛИЗЕ У КОНТАКТНОЈ МЕХАНИЦИ

Резиме:

Контактна механика има важну улогу у бројним инжењерским проблемима и једна је од изазовнијих тема у нумеричким процедурама усљед присуства изражених нелинеарности. У овом раду дат је преглед контактних формулација развијених у оквиру изогеометријске анализе (ИГА). Након приказа класичних приступа у оквиру методе коначних елемената и њихових ограничења, представљени су основни појмови Б-сплајнова и НУРБС функција, као и кратак преглед примјене изогеометријске анализе у механици. Затим су размотрене контактне формулације у оквиру ИГА, при чему је показано да овакви приступи омогућавају већу тачност и рачунску ефикасност у поређењу са класичним методама коначних елемената.

Кључне ријечи: ИГА, НУРБС, контактна механика, МКЕ, контактни алгоритми

1. INTRODUCTION

In mechanical and structural systems, the interaction between bodies that come into contact may play a crucial role in determining system behavior. The characteristics of contact interactions strongly influence the mechanical response of structures. Consequently, the development of efficient and accurate computational methods for capturing these effects remains one of the major challenges in computational contact mechanics. Contact mechanics has a wide range of applications across various industrial sectors, such as the automotive, marine and aerospace industries. Accurate prediction of contact pressures for machine elements plays an important role for reliable and efficient design. In structural and civil engineering, contact mechanics is important for analyzing joints, interfaces, and foundations, including soil-structure interaction. In mechanical engineering, applications include the design of gears and bearings, as well as problems that occur in metal forming. It has significant role in biomechanics as well. Together, these applications demonstrate that contact mechanics is one of the essential tools, and without it, understanding and predicting the mechanical behavior would be difficult.

The scientific study of contact problems dates back several centuries. In the fifteenth century, Leonardo da Vinci observed that the friction force is proportional to the weight of sliding blocks. This observation was later formulated as a physical law by Amontons in 1699 and subsequently extended by Coulomb in 1781, [1]. The theory of elasticity was applied to contact mechanics in the nineteenth century, when Heinrich Rudolf Hertz studied elastic contact between two spheres, [2]. The general problem of equilibrium of a linear elastic body in contact with a rigid, frictionless foundation is introduced in [3], and the first rigorous analysis in [4] led to the formulation of variational inequalities.

In addition to analytical approaches, which serve as the foundation for addressing contact problems, the application of numerical modeling provides a much broader possibility. In this context, the finite element method (FEM) has become the most widely used computational framework for the analysis of contact problems. Detailed studies on the implementation of contact conditions and constraints in finite element analysis (FEA) can be found in [5], [6], [7], [8]. However, contact problems based on classical FEA often suffer from strong nonlinearities, lack of smoothness, and ill-conditioning, which often lead to significant oscillations and high jumps in the contact response, [9].

Motivated by these limitations, alternative numerical approaches have been actively explored. One such approach is isogeometric analysis (IGA), which was first introduced in [10], [11] to link computer aided geometric design (CGAD) directly to FEA. Using the B-splines and non-uniform rational B-splines (NURBS), it is possible to represent the geometry exactly, and to achieve smooth representation of unknown fields with increased interelement continuity.

The aim of this paper is to provide an overview of the main developments in contact problems formulated within the isogeometric framework. The structure of the paper is organized as follows. Section 2 provides an overview of contact modeling within the classical FEM. To establish the necessary foundations for isogeometric formulations, the subsequent section briefly introduces B-splines and NURBS. Section 4 reviews the fundamentals of IGA, and Section 5 presents some of the isogeometric contact formulations and their applications.

2. FEM CONTACT ANALYSIS

Contact mechanics is applied to a wide range of one-, two- and three-dimensional problems. There are several methods for enforcing contact constraints that prevent physical penetration between interacting bodies, such as: Lagrange multiplier method, Penalty method, Nitsche method, barrier method, augmented Lagrange methods and perturbed Lagrange formulation, [8]. Frictional contact represents the general case, which can be simplified by not taking the friction into account, leading to a frictionless formulation. The Lagrange multiplier method enforces contact constraints exactly through additional variables, providing high accuracy at the cost of increased system complexity. In contrast, the penalty method introduces no additional variables, but the constraint conditions are only satisfied approximately. As the penalty parameter increases toward infinity, the solution of the Lagrange multiplier method is formally recovered. However large values of the penalty parameter can cause severe ill-conditioning of the system. This limitation can be overcome by employing the augmented Lagrangian method that combines the penalty and Lagrange multiplier methods, [12].

Beyond constraint enforcement, the formulation and discretization of the contact interface play a crucial role in the accuracy and robustness of FEM contact simulations. One of the earliest discretization strategies for small deformation problems is the node-to-node (NTN) strategy [13].

For the case when the nodes are arbitrarily distributed, the segment approach for contact discretization is suggested in [14]. The node-to-segment (NTS) approach is widely used in large deformation contact problems, with the implementations reported in [15], [16], [17]. This method, however, doesn't fulfill the patch test that ensures that the discretization error at the contacting surfaces decreases with mesh refinement. Several studies have been conducted to investigate this issue [18], [19]. Another discretization approach that overcomes this issue is segment-to-segment (STS) that usually employs the so-called intermediate contact surfaces, see [20], [21]. The Gauss-point-to-surface (GPTS) offers a computationally efficient alternative, as it does not require segmentation for evaluating the contact integral. Mortar methods satisfy the contact patch test and provide a consistent formulation but require a high computational cost [22], [23]. The change in boundary conditions in contact problems can cause a loss of regularity of the solution. In such cases, standard shape functions are unable to represent the resulting kinks, leading to oscillatory numerical solutions. The comparison of the classical *h*-, *p*-, *hp*-, and *rp*-version FEM formulations on the Hertzian contact problem can be found in [24].

Significant effort has been devoted to contact problems involving beam structures within the classical FEM framework. Frictional contact between beams in three-dimensional space undergoing large displacements is investigated in [25]. Frictionless contact formulations for three-dimensional beams with rectangular cross-sections, allowing large displacements under the assumption of small strains, were developed in [26]. Several geometrical situations of contacting bodies, such as surface-to-surface, curve-to-surface, point-to-surface, curve-to-curve, point-to-curve, point-to-point are investigated in [27]. Curve-to-solid beam element is proposed in [28] and it combines the advantages of the surface contact and the flexibility of the application to beam-to-beam contact. Penalty formulation for shear-deformable beams with elliptical cross-sections, that enables line or area contact descriptions instead of purely point-wise interactions is investigated in [29]. An enhanced point-wise beam-to-beam contact formulation addressing cases with very acute contact angles was proposed in [30]. A geometrically exact theory for contact interactions of one-dimensional manifolds was proposed in [31], providing a consistent formulation applicable to various finite element beam models. Enhanced multiple-point beam-to-beam contact formulations with improved robustness and accuracy were introduced in [30]. Contact-friction modeling within assemblies of elastic beams was further advanced by [32] with applications to complex problems such as knot tightening. A novel non-material kinematic description was introduced in [33], where the mixed finite element formulation employs a compound coordinate system consisting of a transversal material coordinate and a spatial contour coordinate, which is particularly well-suited for the analysis of axially moving structures. A geometrically nonlinear steady-state formulation for a slack two-pulley belt drive was derived in [34], using a mixed Eulerian-Lagrangian finite element scheme, with validation against obtained semi-analytic solution. Furthermore, a Kirchhoff-Love shell stress resultant plasticity model incorporating membrane effects was proposed in [35] and applied to efficient roll forming simulations. A finite element formulation for line-to-line contact interaction of thin beams with arbitrary orientation was developed by [36], significantly improving robustness and consistency for slender beam contact problems. As point contact formulations are not applicable for small contact angles, a novel all-angle beam contact formulation was proposed in [37], that employs a point contact approach in the range of large contact angles and a line contact approach in the range of small contact angles.

Despite their widespread use, classical FEM-based contact formulations suffer from several limitations, particularly related to geometry approximation, sensitivity to mesh discretization, and difficulties in enforcing contact constraints consistently. These issues motivate the development of alternative numerical frameworks that offer smoother geometry representations and improved robustness, such as isogeometric analysis.

3. B-SPLINE AND NURBS

This section provides a brief overview of B-Splines and NURBS, introducing the main definitions and characteristics. For a more comprehensive discussion, including detailed definitions, mathematical formulations, and properties, refer to [10], [38].

B-Spline curves are polynomial functions defined as a linear combination of *n* basis functions. A curve of order *p* is defined as:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} B_{i,p}(\xi)\mathbf{P}_i \tag{1}$$

Here, $\mathbf{P}_i$ is the i-th control point and $B_{i,p}$ is the associated basis function. Basis functions are defined by a knot vector $\xi=\{\xi_1, \xi_2, \ldots, \xi_i, \ldots, \xi_{n+p+1}\}$ and are found recursively, using the Cox-de Boor algorithm. An example of a B-Spline curve is shown in Figure 1, along with its representation in the parameter and parent spaces. B-Spline surfaces are defined analogously and given as:

$$\mathbf{S}(\xi,\eta)=\sum_{i=1}^{n}\sum_{j=1}^{k}N_{i,p}(\xi)M_{j,q}(\eta)\mathbf{P}_{ij} \tag{2}$$

where $N_{i,p}$ and $M_{j,q}$ are basis functions of B-Spline curves defined over a knot vectors $\xi=\{\xi_1, \xi_2, \ldots, \xi_i, \ldots, \xi_{n+p+1}\}$ and $\eta=\{\eta_1, \eta_2, \ldots, \eta_i, \ldots, \eta_{k+q+1}\}$.

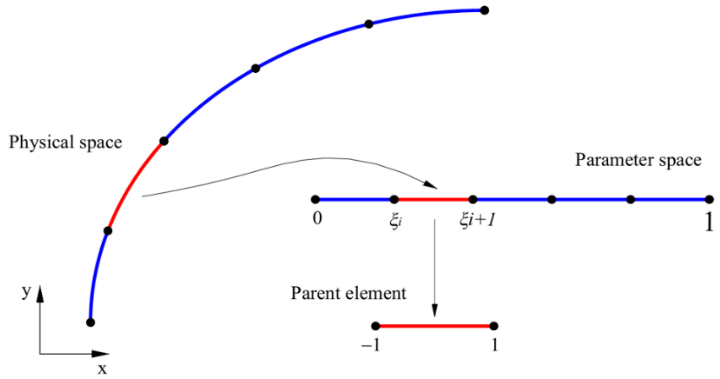

Figure 1. Physical space, parameter space and parent element, adapted from [39].

B-Spline basis functions are non-negative and form a partition of unity for each value of parameter ξ. Their continuity is governed by the knot vector, and in general, they have C^{p-1} continuity at the knots. If a knot has multiplicity m, the continuity is reduced to C^{p-m}. B-Spline basis functions also have local support property, as each basis function of order p is non-zero over p+1 knot spans.

Refinement strategies analogous to the finite element h- and p- can be employed through knot insertion and order elevation techniques. These techniques can be applied without modifying the underlying geometry or its parameterization. Another strategy of order elevation is referred to as k-refinement, and it uses the fact that the processes of knot insertion and order elevation do not commute. This concept is illustrated in Figure 2. for a B-Spline basis of order p=2. When knot insertion is performed first and followed by order elevation, the resulting basis functions are C^1 continuous at the knots. In contrast, if the order is elevated first and knot insertion is applied afterward, the continuity is increased to C^2. In both cases, the polynomial order is increased, however within k-refinement, it results in the smaller number of basis functions and associated control points, as well as in increased continuity.

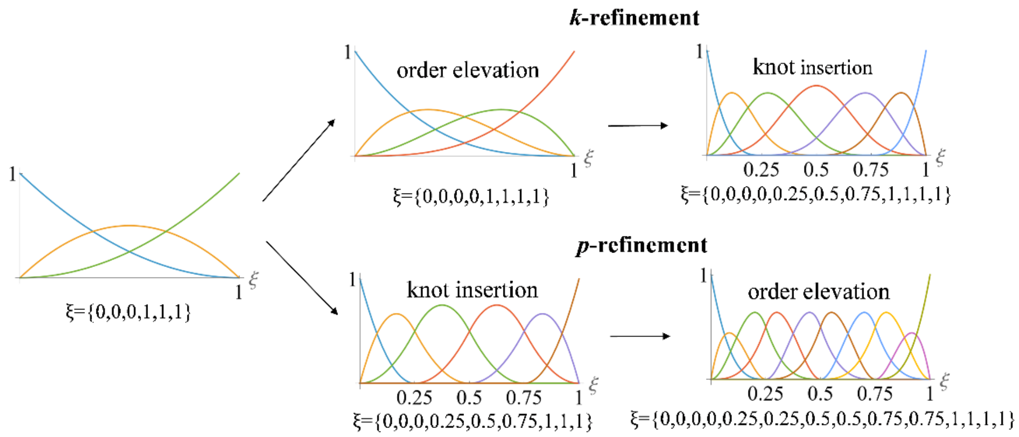

Figure 2. Higher order B-Spline basis functions (drawing by author).

Although versatile, polynomial B-splines cannot exactly represent conic geometries, including circles and ellipses. This issue is addressed by more general non-uniform rational B-Splines. A NURBS curve is defined as:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{P}_i, \quad R_{i,p} = B_{i,p}(\xi)w_i \Big/ \sum_{j=1}^{n} B_{j,p}(\xi)w_j \tag{3}$$

where $R_{i,p}$ are the rational basis functions, and w_i are the weights associated with the i-th control point. The rational basis functions have the same properties as B-spline basis functions.

A detailed comparison of classical and isogeometric parameterizations can be found in [9]. Due to their inherent properties, B-spline and NURBS functions provide a suitable basis for advanced numerical analysis frameworks.

4. ISOGEOMETRIC ANALYSIS

Within the IGA framework, spline-based basis functions are employed to represent geometry exactly while simultaneously approximating the unknown solution fields. By using higher-order, highly continuous basis functions, IGA offers several advantages over classical finite element analysis, including improved numerical accuracy for a given number of degrees of freedom. This section provides a brief overview of relevant literature in structural mechanics. Certain mechanical models, such as Bernoulli-Euler beams and Kirchhoff-Love shells, require C^1 interelement continuity, which is not readily available in classical Lagrange-based finite element methods. Isogeometric analysis naturally provides this higher-order continuity and has therefore been widely applied to a broad range of structural mechanics problems in static analysis. Figure 3 presents a representative example of a rod in space with a circular cross-section, whose centerline is parameterized by cubic B-spline together with the associate basis functions defined over the knot vector ξ={0,0,0,0,0.25,0.5,0.75,1,1,1,1} with C^2 interelement continuity.

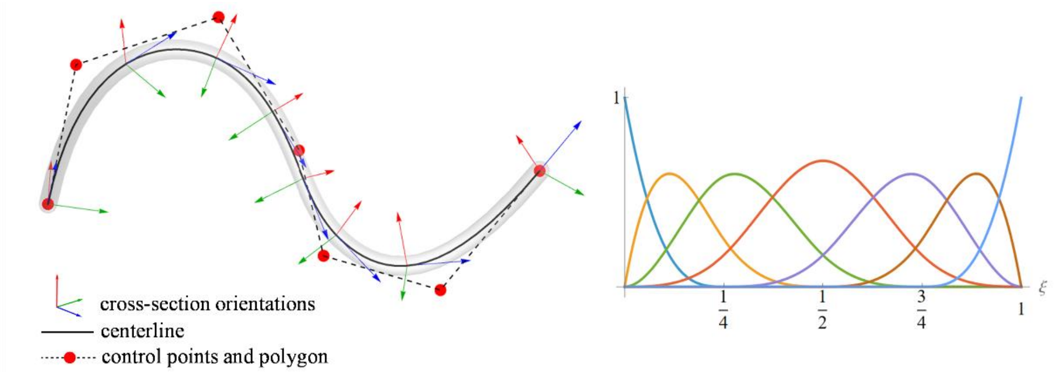

Figure 3. Isogeometric parameterization of a rod with B-Spline curve and cubic basis functions (drawing by author).

In [40], a B-spline interpolation approach and an effective parameterization of the geometric torsion were proposed for Kirchhoff-Love space rods, demonstrating high accuracy. Rotation-free isogeometric formulations for arbitrarily curved Bernoulli-Euler beams were developed in [41], and shown to be applicable also to strongly curved beams. The theoretical background follows the formulation presented in [42]. The treatment of finite rotations and large deformations can be found in [43]. An efficient collocation-based approach for Bernoulli-Euler beams and Kirchhoff plates was presented for the first time in [44]. Another application of isogeometric collocation methods to Cosserat rod models and rod structures was discussed in [45], where a mixed formulation alleviating shear locking in thin rods was presented.

Geometrically nonlinear and multi-patch isogeometric formulations for spatial Euler-Bernoulli beam structures were developed in [46], and invariant formulations for three-dimensional Kirchhoff rods were proposed in [47]. A displacement-free isogeometric formulation for the Timoshenko beam problem was presented in [48], which is completely locking-free and involves a reduced number of degrees of freedom. Several studies have addressed numerical locking phenomena within isogeometric formulations. Locking-free strategies for curved and thick beam structures were proposed in [49], while shear locking in Timoshenko beams with mixed and displacement-based formulations was specifically addressed using isogeometric collocation techniques in [50]. In [51], a blended mixed B-Spline formulation was introduced to effectively eliminate membrane locking in plane curved Kirchhoff rods. A novel isogeometric collocation method for geometrically exact Cosserat rods was proposed in [52] to avoid locking under large deformations and rotations by employing a mixed Hellinger-Reissner formulation.

Isogeometric analysis has also been widely applied to vibration and dynamic analysis of structural systems. Early studies demonstrated the effectiveness of IGA for structural vibration problems compared to classical FEM, particularly using the *k*-refinement, as shown in [53]. Nonlinear vibration analyses of Euler-Bernoulli beams were investigated in [54], while vibration analysis of free-form curved beams using curve reparameterization techniques was presented in [55]. A rotation-free NURBS-based formulation for in-plane vibrations of arbitrarily curved Bernoulli-Euler beams was developed in [39], with detailed studies on curvature effects, *hpk*-refinement, and convergence behavior. NURBS-based vibration analysis of laminated Timoshenko deep-curved beam beams was discussed in [56], while the novel higher order formulations for improved vibration accuracy are proposed in [57]. In-plane and out-of-plane vibrations of Timoshenko beams are studied in [58]. Applications to variable stiffness laminated composite beams were reported in [59]. Nonlinear static isogeometric analysis of arbitrarily curved Kirchhoff-Love shell was presented in [60], where the accuracy of the proposed method was also demonstrated for strongly curved shells. Isogeometric collocation formulation for Reissner-Mindlin shell problem was developed in [61], avoiding locking by employing high polynomial degrees. A hierarchic family of isogeometric shell finite elements was introduced in [62], while in [63] shear-deformable and rotation-free shell formulations were proposed, eliminating the observed artificial oscillations in transverse shear forces. Galerkin-based shell formulations addressing locking effects through higher-order elements were proposed in [64]. Isogeometric formulations have also been applied to plate structures, including stability analysis of laminated plates and vibration analysis of Kirchhoff plates using higher-order mass matrices [65], [66].

Overall, the reviewed studies clearly demonstrate the versatility and effectiveness of isogeometric analysis across a broad range of structural mechanics problems. The combination of exact geometry representation, higher-order continuity, and flexible refinement strategies makes IGA a powerful tool for the analysis of beams, rods, shells, and plates.

5. ISOGEOMETRIC CONTACT ANALYSIS

Due to its features, isogeometric analysis has found widespread application in computational mechanics, particularly in contact mechanics, where the mechanical response is characterized by strong nonlinearities. This section provides an overview of isogeometric analysis formulations developed for contact mechanics.

Significant progress has been made in the development of IGA-based contact formulations for beams and rods. An isogeometric collocation method for frictionless contact of spatial rods was introduced in [67], where contact penalty forces are applied as point loads within the collocation scheme. The formulation demonstrated robustness and accuracy in challenging applications such as knot tightening and woven mesh structures. Dynamic extensions were subsequently proposed in [68], where an isogeometric collocation method based on the geometrically exact Cosserat rod model was developed for nonlinear dynamic analysis, including viscoelastic damping and frictional contact. A geometrically exact isogeometric beam formulation suitable for nonlinear static and dynamic simulations with collisions and frictional contact was presented in [69]. An efficient and robust simulation of contact is achieved through the adoption of a non-smooth dynamics formulation based on differential-variational inequalities.

Further developments include an isogeometric beam contact formulation based on Cosserat rod kinematics with impenetrability constraints, employing a surface-to-surface penalty contact algorithm and achieving accurate contact pressures with fewer degrees of freedom than brick element models [70]. To address limitations of curve-to-curve contact formulations based on closest point projections, a curve-to-solid beam contact algorithm was proposed in [28], combining the accuracy of surface contact descriptions with the flexibility of beam-to-beam formulations. In addition, a pointwise beam-to-beam contact formulation based on C^1-continuous spline contact elements was introduced in [71], alleviating contact singularities caused by sharp element corners and ensuring a smooth contact description.

For plate and shell structures, several IGA-based contact discretizations have been developed. A bilateral isogeometric collocation contact formulation was proposed in [72], where stress oscillations arising from higher continuity can be avoided through knot relocation and repetition. A weighted point-based method (PTS+) was introduced in [73], enabling stable collocation of virtual contact work comparable to mortar methods, while significantly reducing the number of contact evaluation points.

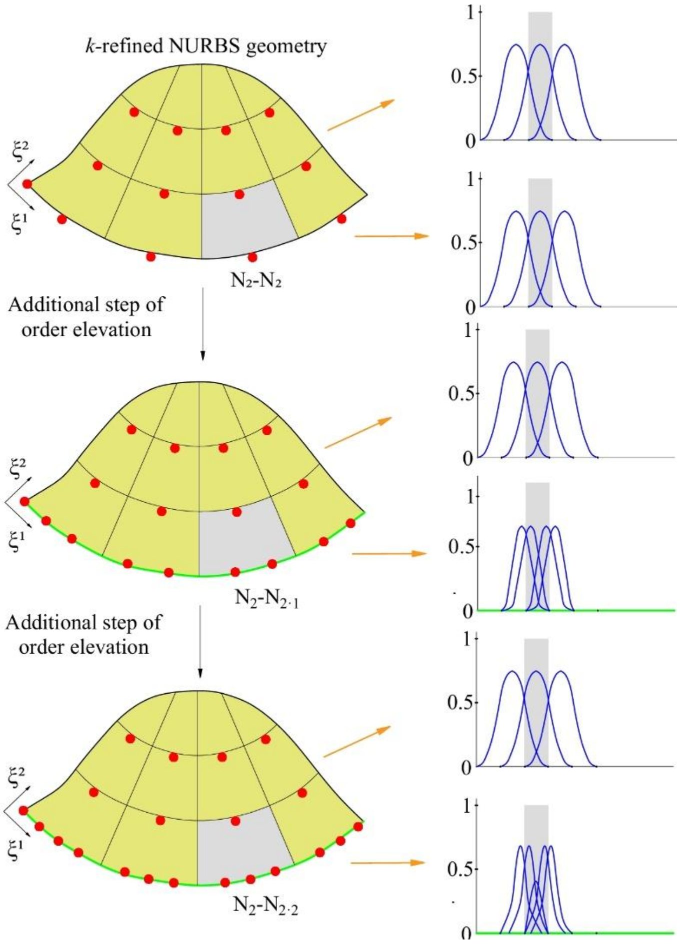

Figure 4. An illustrative application procedure of additional steps of order elevation strategy to the contact boundary layer, adapted from [74].

IGA contact formulations based on Gauss-point-to-surface (GPTS) and mortar techniques have also received considerable attention. A patch-wise contact detection strategy combined with robust surface-to-surface frictionless contact and corner smoothing was proposed in [75]. NURBS-based isogeometric contact formulation with superior accuracy, and convergence behavior compared to classical C^0-continuous FEM is demonstrated in [76]. A mortar-based NURBS formulation for Coulomb frictional contact under large deformations was presented in [77]. Mortar IGA formulations demonstrated significantly improved smoothness of traction history curves compared to classical FEM in large deformation and large sliding contact problems [78]. A three-dimensional mortar formulation for a frictional contact under finite deformations, yielding non-negative contact pressures, reduced oscillations, and improved smoothness of global contact forces and moments was introduced in [79]. Preserving the exact NURBS geometry at the contact interface, a mortar-based isogeometric contact scheme was shown to outperform classical FEM particularly for curved contact problems [80]. A collocated contact surface (CCS) formulation was proposed in [81], combining collocation on the contact surfaces with a Galerkin treatment of the bulk, achieving patch-test consistency and accurate results in both small and large deformation regimes. A symmetry-preserving two-half-pass isogeometric formulation coupled with a Gauss-point-to-surface discretization and penalty regularization further demonstrated robust performance for large deformation frictionless contact [82]. From a broader perspective, isogeometric mortar methods have been investigated from both theoretical and numerical viewpoints, consistently showing superior stability, accuracy, and robustness compared to conventional FEM-based contact formulations [83].

To improve both efficiency and accuracy in isogeometric contact analysis, a varying-order NURBS discretization strategy was proposed in which higher-order NURBS are applied locally at the contact interfaces, while lower-order discretizations are retained in the bulk domain, resulting in enhanced accuracy on coarse meshes and reduced computational cost [74]. With very smooth basis functions it is not possible to accurately capture the sharp changes in the distribution of contact pressure, as shown in [74]. An order elevation of the C^1 continious contact layer is illustrated in Figure 5. which led to the shrinking of the multi-knot span support of the basis functions and allowing the local changes in contact pressure to be captured more accurately. Since standard NURBS-based IGA lacks local refinement capabilities. To address this issue, a local refinement algorithm for analysis-suitable T-splines was developed in [84], enabling adaptive refinement while preserving the exact geometric representation required for reliable contact simulations. A T-spline-based isogeometric formulation for frictionless contact demonstrated superior accuracy compared to both uniform and non-uniform NURBS discretizations through comprehensive convergence studies [85]. Due to local refinement property, it is possible to achieve larger number of elements in the vinicinity of the contact surface. Figure 5. shows the two examples of a mesh for the Hertz contact problem.

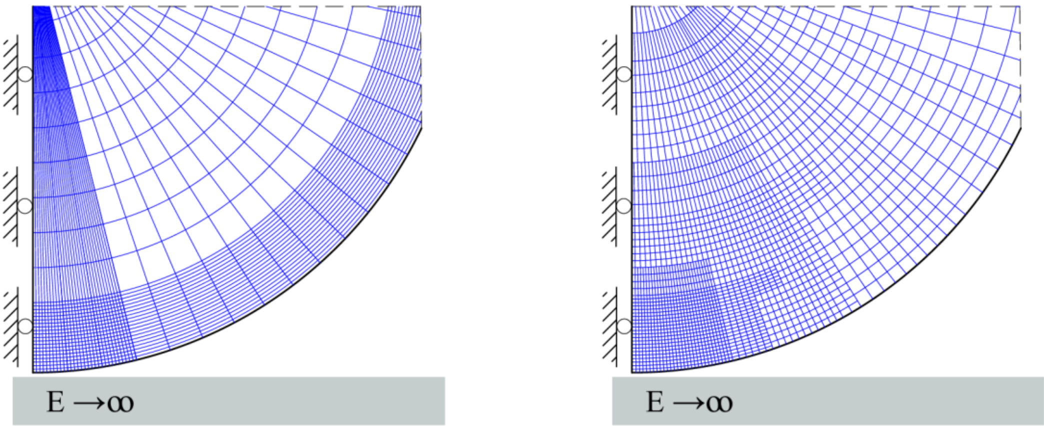

Figure 5. Different meshes for a Hertz contact problem, adapted from [86].

An alternative local refinement strategy based on locally refined non-uniform rational B-Spline (LR NURBS) was introduced for contact analysis of three-dimensional solids and membranes, achieving higher accuracy at lower computational cost compared to uniform refinement while maintaining exact geometry representation [87]. Hierarchical NURBS-based isogeometric contact algorithms were developed in [88], featuring hierarchical mortar discretizations for frictional contact, partition-of-unity mortar bases, and both exact and regularized constraint enforcement strategies. The varying-order concept was further extended to three-dimensional large-deformation frictional contact problems, demonstrating enhanced accuracy and computational efficiency [89]. A two-half-pass isogeometric mortar contact formulation incorporating extended finite element interpolation was proposed, eliminating the need for segmentation at overlapping master–slave element boundaries, while accurately capturing pressure discontinuities and achieving robust, patch-test-consistent results [90].

Besides the classical contact approach based on phenomenological observations (condition of zero penetration), it is possible to model contact by considering fundamental physical steric forces. These are volumetric repulsive forces that are often modeled in combination with attractive van der Waals forces. The first such model for beam-beam interaction is developed in [91]. Authors have introduced a concept of section-section interaction potential which reduces 6D integral over volumes of both beams to pre-calculated section-section potential and 2D numerical integration. The method is further improved by ameliorating the section-section law, using the isogeometric paradigm, and considering the complex dynamic simulations [92], [93], [94], [95].

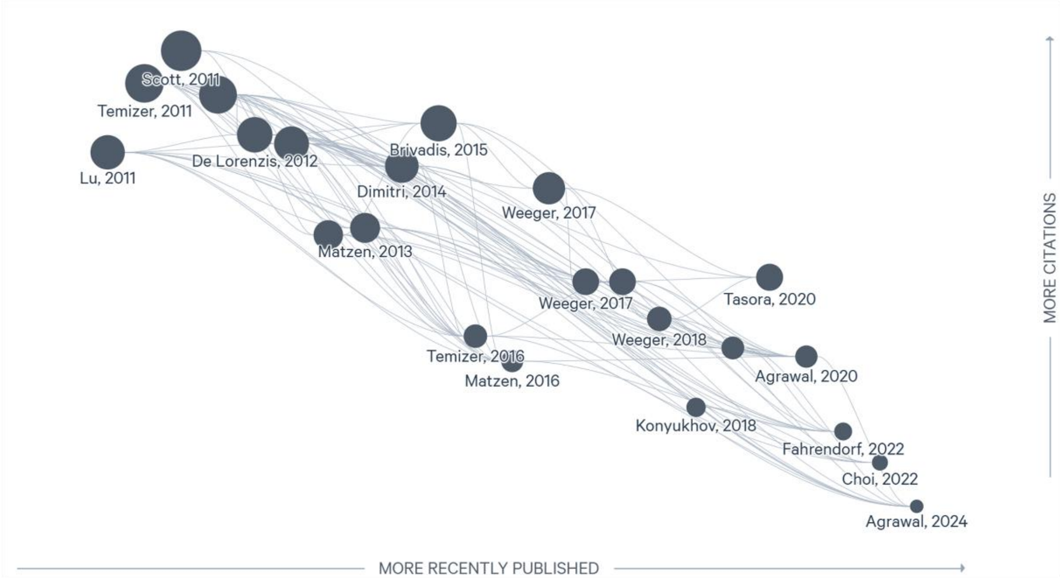

Figure 6. Citation map created by [96].

Figure 6. presents a citation network generated using Litmaps, illustrating representative isogeometric contact formulations and their chronological development from early NURBS-based approaches to recent advances. The studies reviewed in this section clearly demonstrate that isogeometric analysis provides a powerful and flexible framework for the numerical treatment of contact problems. By exploiting exact geometry representation, higher-order continuity, and locally refined spline technologies, isogeometric contact approaches consistently outperform classical FEM formulations in terms of accuracy, robustness, and smoothness of the contact response. At the same time, recent developments have shown that these advantages can be achieved with smaller computational costs.

6. CONCLUSIONS

This paper provides an overview of isogeometric contact formulations, covering the foundations of contact problems as well as recent developments and applications within the isogeometric analysis framework. The reviewed studies demonstrate that isogeometric contact approaches can achieve higher accuracy and improved computational efficiency compared to conventional finite element methods, largely due to the use of smooth, higher-continuity spline-based basis functions for representing both geometry and unknown fields. These characteristics make isogeometric analysis particularly attractive for contact problems involving strong nonlinearities.

Despite extensive research efforts, there are still some challenges to be addressed. Recent advances in NURBS-based discretization techniques, including locally refined and adaptive approaches, have demonstrated promising performance in mechanical contact problems. T-spline-based isogeometric analysis offers advantages over standard NURBS formulations due to its inherent local refinement capabilities, and further development of adaptive refinement strategies could enhance its effectiveness in contact simulations. Collocation-based isogeometric formulations, as well as mortar formulations, have shown improved robustness and flexibility. All these approaches are in the early stage of development and continued advancements in isogeometric analysis are expected to expand its applicability as a powerful and reliable tool for complex contact simulations.

ACKNOWLEDGMENTS

This research was partly funded by the Austrian Science Fund (FWF) 10.55776/P36019. For the purpose of open access, the authors have applied a CC BY public copyright licence to any Author Accepted Manuscript version arising from this submission.

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