05
Research paper I Оригинални научни рад
DOI 10.7251/STP2215045C
ISSN 2566-4484

Anđelko Cumbo, University of Banja Luka, andjelko.cumbo@aggf.unibl.org
Gordana Broćeta, University of Banja Luka, gordana.broceta@aggf.unibl.org
Marina Latinović Krndija, University of Banja Luka, marina.latinovic@aggf.unibl.org
Mladen Sljepčević, University of Banja Luka, mladen.slijepcevic@aggf.unibl.org
Žarko Lazić, University of Banja Luka, zarko.lazic@aggf.unibl.org

THE INFLUENCE OF CONCRETE VISCOUS DEFORMATIONS DURING BEHAVIOR CALCULATION OF CABLE-STAYED BRIDGES

Abstract

On the calculation example of a cable-stayed bridge with oblique cables, it was pointed out the shrinkage and creep of concrete significantly affects the distribution of stresses and deflections of the span beam over time. The segmental type of bridge construction is considered, which implies the successive introduction of concrete rheology, for cases of controlled and free beam deflections during construction, i.e. with or without cable tightening on previously performed segments. The phenomenon of reduced sensitivity of the span beam to the effects of concrete rheology is pointed out, in case of controlled deflections by cable tightening, while without cable tightening the deflections increase over time and can negatively affect the usability of the structure. The analyses were performed using an appropriate algorithm (developed by the author) which introduced composite layered finite elements with viscous properties into the calculation.

Keywords: cable-stayed bridges, concrete creep and shrinkage, finite elements.

УТИЦАЈ ВИСКОЗНИХ ДЕФОРМАЦИЈА БЕТОНА ПРИ ПРОРАЧУНУ ПОНАШАЊА МОСТОВА СА КОСИМ ЗАТЕГАМА

Сажетак

На примјеру прорачуна овјешеног моста (са косим кабловима) указано је да скупљање и течење бетона значајно утиче на расподјелу напрезања и угиба распонске греде током времена. Разматран је сегментни тип градње моста, што подразумијева сукцесивно увођење реологије бетона, за случајеве контролисаних и слободних угиба греде током градње, односно са или без дотезања каблова на претходно изведеним сегментима. Указано је на феномен смањене осјетљивости распонске греде на ефекте реологије бетона ако се контролишу угиби дотезањем каблова, док се без дотезања каблова угиби током времена повећавају и могу негативно утицати на употребљивост конструкције. Анализе су урађене коришћењем одговарајућег алгоритма (развијеног од стране аутора) којим су у прорачун уведени спрегнути слојевити коначни елементи са вискозним својствима.

Кључне ријечи: овјешени мостови, течење и скупљање бетона, коначни елементи.

1. INTRODUCTORY REMARKS

It is known that the shrinkage and creep of concrete in time can have a significant impact on the changes in stress and strain in composite structures [1] [2] [3] [4]. These deformations can be two to three or even several times larger than elastic deformations, which is why it is very important to perform adequate analysis when designing structures, taking into account the degrees of indeterminacy of the structure and different time intervals of inclusion of individual elements/layers in stress activity (segmental construction).

To solve such and similar problems, a computational algorithm with layered finite elements was developed (by the author) within the broader work on the analysis of the influence of rheological properties of concrete and prestressed reinforcement in complex (statically indeterminate) composite structures. To gain the stiffness matrix of finite element (FE), the layer method in cross-section was applied, and influences due to viscous properties of the material were introduced via fictitious load [1] [2]. By applying an incremental form of stress-strain relation for individual materials it is enabled to generalize the procedure for discontinuous and continuous changes by introducing the required number of fictitious ($\Delta t_k=0$) and finite time intervals ($\Delta t_k \neq 0$). This reduces the overall calculation procedure to solving algebraic equations (in matrix form), which enables the calculation automatization, thus an appropriate software algorithm is formed, suitable for application in practice when it comes to controlling the serviceability limit states.

In general, the calculation model includes rigidly composed layered elements, and applies the *Bernoulli* hypothesis of flat cross-sections and the linear theory of concrete creep. The expressions introduce the following labels for common materials: a-structural steel, c-concrete, s-reinforcement, and p-prestressing cables. In the general case, for the deformation of the observed fiber in the composed cross-section in the current time interval Δt_k, the equality is given [1]:

$$\Delta\varepsilon_k=\Delta\varepsilon_{r,k}+\Delta\kappa_k\cdot y \tag{1}$$

where:

$\Delta\varepsilon_k$ - deformation increment of the observed fiber in the cross-section in *k*-th time interval,

$\Delta\varepsilon_{r,k}$ - deformation increment at the level of the reference axis of the cross-section *r* (y=0),

$\Delta\kappa_k$ - cross-sectional curvature,

y - distance of the observed fiber from the reference axis *r*.

According to the linear distribution of deformations, the stresses along the height of the composed cross-section are also distributed linearly. However, unlike deformations, the stress increment for each layer/material of the composite cross-section is defined with a separate pair of parameters $\Delta\alpha_{r,k}$ and $\Delta\beta_k$, following the constitutive stress-strain relations for individual materials, whereby abrupt changes occur at the contacts of different materials. If the expressions are written in vector form, equations for individual materials in the current time interval Δt_k have the following forms:

$$\begin{Bmatrix}\Delta\alpha_r\\ \Delta\beta\end{Bmatrix}_{m,k}=E_m\cdot\begin{Bmatrix}\Delta\varepsilon_r\\ \Delta\kappa\end{Bmatrix}_{m,k}\ ;\ m=a,s \tag{2}$$

$$\begin{Bmatrix}\Delta\alpha_r\\ \Delta\beta\end{Bmatrix}_{c,k}=E_{c(k,k-1)}\cdot\left(\begin{Bmatrix}\Delta\varepsilon_r\\ \Delta\kappa\end{Bmatrix}-\begin{Bmatrix}\Delta\varepsilon_r^*\\ \Delta\kappa^*\end{Bmatrix}\right)_{c,k} \tag{3}$$

$$\begin{Bmatrix}\Delta\alpha_r\\ \Delta\beta\end{Bmatrix}_{p,k}=E_p\cdot\begin{Bmatrix}\Delta\varepsilon_r\\ \Delta\kappa\end{Bmatrix}_{p,k}+\begin{Bmatrix}\Delta\bar{\sigma}_{pr}\\ 0\end{Bmatrix}_{p,k} \tag{4}$$

The vector of the free deformations increment in concrete, when the shrinkage deformation is equal for all points along the height of the cross-section, has the form:

$$\begin{Bmatrix}\Delta\varepsilon_r^*\\ \Delta\kappa^*\end{Bmatrix}_{c,k}=\sum_{i=1}^{k-1}\frac{1}{E^*_{c(k,i-1)}}\cdot\begin{Bmatrix}\Delta\alpha_r\\ \Delta\beta\end{Bmatrix}_{c,i}+\begin{Bmatrix}\Delta\varepsilon_n\\ 0\end{Bmatrix}_{c,k} \tag{5}$$

The basic equation of the composed FE [1] in the local coordinate system for the current time interval Δt_k is:

$$[K]_k\cdot\{\Delta q_r\}_k=\{\Delta Q\}_k-\{\Delta Q^*\}_{c,k}-\{\Delta Q^*\}_{p,k} \tag{6}$$

where:

$[K]_k$ - stiffness matrix of the composed FE,

$\{\Delta q_r\}_k$ - vector of nodal displacements from the reference axis *r*,

$\{\Delta Q\}_k$ - vector of external nodal forces,

$\{\Delta Q^*\}_{c,k}$ - vector of fictitious nodal forces due to creep and shrinkage of concrete,

$\{\Delta Q^*\}_{p,k}$ - vector of fictitious nodal forces due to relaxation of prestressed reinforcement.

Forming a system of equations for a total FE mesh of structure, requires the prior formation of the basic equations for each FE. Thereby, it is necessary to transform the stiffness matrix and force vectors for each FE from the local to the global coordinate system. The equilibrium equation of the system (matrix shape) is obtained when the stiffness matrices and force vectors from expression (6) for each FE are superimposed in accordance with the connection criterion for system nodes. By setting the stiffness matrix coefficients and the FE force vectors at the appropriate positions, a generalized equilibrium equation is formed for the whole composite system, for the current time interval Δt_k:

$$[\tilde{K}]_k \cdot \{\Delta \tilde{q}_r\}_k = \{\Delta \tilde{Q}\}_k - \{\Delta \tilde{Q}^*\}_{c,k} - \{\Delta \tilde{Q}^*\}_{p,k} \tag{7}$$

The system of algebraic equations (7) includes the elastic and rheological properties of the applied materials by layers, for all individual FEs in the construction system. Visco-elastic changes are included in finite time intervals ($\Delta t_k \neq 0$), while elastic (discontinuous) changes are included in fictitious time intervals($\Delta t_k = 0$). The connection between the displacement vector and the force vector in the system nodes is established via the stiffness matrix. By solving the system of algebraic equations (7) the vector of nodal displacements of the system for the current time interval Δt_k is determined with the previous introduction of boundary conditions. After the transformation of the displacement vector into local coordinate systems, the component deformations for each FE are determined [1]:

$$\begin{Bmatrix} \Delta\varepsilon_r \\ \Delta\kappa \end{Bmatrix}_k = [B_r] \cdot \{\Delta q_r\}_k + \begin{Bmatrix} \Delta\varepsilon_N \\ -\Delta\kappa_M \end{Bmatrix}_k \tag{8}$$

where:

$\Delta\varepsilon_{N,k}$ - part of deformation resulting from averaging of fictitious normal forces $\Delta N_{N,k}$ of the observed element,

$\Delta\kappa_{M,k}$ - the part of the curve originating from the external distributed load in the middle-span of the element introduced through the equivalent nodal forces, where the moments $\Delta M_{M,k}$ appear, which do not exist,

$[B_r]$ - interpolation matrix (shape function) for the reference axis in the nodes of the element.

The parameters of total stresses and strains for the discrete moment are determined by the superposition of the previous state and the change of state in the current time interval (step-by-step procedure: $t_k = t_{k-1} + \Delta t_k$):

$$\begin{Bmatrix} \alpha_r \\ \beta \end{Bmatrix}_k = \begin{Bmatrix} \alpha_r \\ \beta \end{Bmatrix}_{k-1} + \begin{Bmatrix} \Delta\alpha_r \\ \Delta\beta \end{Bmatrix}_k \; ; \; \begin{Bmatrix} \varepsilon_r \\ \kappa \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_r \\ \kappa \end{Bmatrix}_{k-1} + \begin{Bmatrix} \Delta\varepsilon_r \\ \Delta\kappa \end{Bmatrix}_k \tag{9}$$

Stresses in the cross-section of individual layers change linearly by the height, while at the contacts jumps in diagram appear as a result of different material properties. More about that in [2][3][4][5][6][7].

2. CABLE-STAYED BRIDGE CALCULATION EXAMPLE

An example of calculation using the formed algorithm is given for a cable-stayed bridge with steel cables and a concrete girder, with a span 120+120 m. The bridge structure is inhomogeneous and statically indeterminate, with multiples degrees of indeterminacy, where layered viscoelastic elements have been used for the concrete main beam, while elements with elastic properties have been used for the concrete pylon and steel cables. The pylon was considered to have great rigidity and to be rigidly clamped in the ground. The method of cantilever construction was applied, where segments of concrete beams (slabs) are poured on-site and hung with a pair of oblique steel cables on the pylon. This is in accordance with the usual procedures for the construction of suspended structures of medium and large span bridges.

A constant change of the static system during construction is present, and also the changes of concrete in time caused by the shrinkage and creep. This is reflected primarily in the changes in forces in the cables, and in the deflections and stress redistribution in the concrete beam. Thus, it is necessary to estimate the changes in stress and strain, as well as the changes in deflection from the relevant load, as realistically as possible, to prevent negative effects on the load-bearing capacity and serviceability of the bridge.

The analysis of the bridge structure with the introduced rheology of concrete was conducted for a period of 10.000 days (27,4 years), for two different stiffness levels of oblique stay cables. In the first case, the actual stiffness of the cables was taken (no forced deflection/displacement retention, ie s = 0%), which corresponds to the case without the subsequent tightening of previously installed cables when adding each new beam (slab) segment. In this case, the vertical movement (deflection) of the main beam occurs under the actual stiffness of the concrete beam and steel cables. In the second case, the high stiffness of the cables is used in the calculation (s = 100%), ie the horizontality of the bridge beam is maintained by preventing vertical displacements (eg constant tensioning/loosening of cables). In this case, a more realistic assessment of the impact of cable tension on the effects of reduction of concrete deformations in time is possible.

A similar example is analyzed in the paper *Sassone and Casalegno* [5] in which the significant contribution of concrete rheology to the changes of stresses and deformations in the bridge structure over time is pointed out, and the calculation algorithm is implemented in a software environment *Matlab 7* together with a commercial software *TNO Diana 9.4*.

For the example analyzed herein, a schematic model of a suspension bridge with concrete beam, pylon, and steel cables is given in Figure 1. Segmental construction of cantilever beams (slabs) is performed symmetrically starting from the pylon, left and right, by casting on site. At the end of each performed segment, a pair of steel cables are attached, which are used to hang the segments on the pylon. The time of completion of the two symmetric segments is assumed to be 28 days. The length of each segment of the beam (slab) is 24 m, and the height of the pylon at the place of hanging the cables is 50 m. Five segments of beams on each side of the pylon, suspended with five pairs of steel cables were considered.

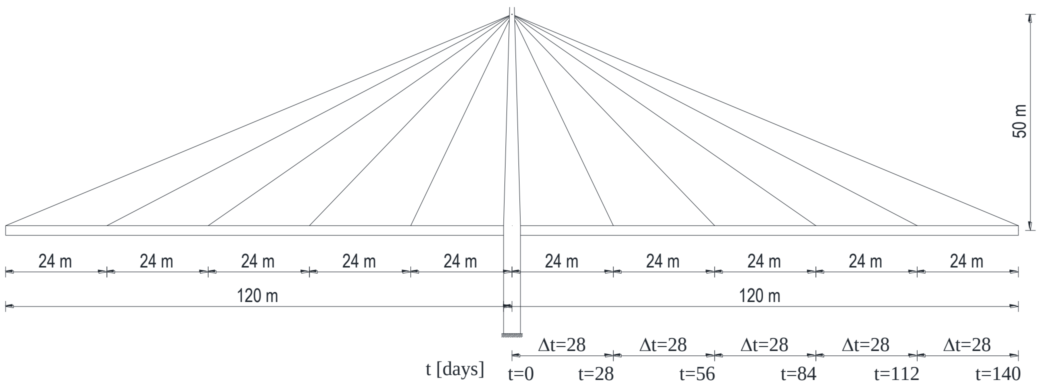

Figure 1. *The scheme of the suspension bridge with oblique cables*

The material characteristics, the predicted loads, and the geometry of the elements are given in Figures 2. Figure 3 shows approximate static systems and loads at characteristic intervals (for easier understanding), while in the calculation model all elements and loads are preicisely entered, and then the activation of loads and stiffness by segments is regulated in accordance with the real conditions of the applied construction technology.

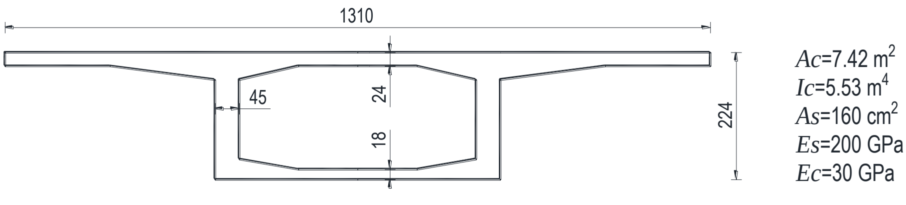

Figure 2. *The cross-section of the main beam of the bridge*

The rheological characteristics of concrete of the main beam were taken following EC2 recommendations, for all relative time relations between discrete moments, and the calculation was carried out by successive application of the AAEM (Age Adjusted Effective Modulus) method. The deformations of the rigid concrete pylon were neglected in the calculation, and also the effect of relaxation within high-grade steel cables over time. A homogeneous concrete beam (slab) with the constant creep and shrinkage properties over beam height was considered. However, as stated, the different ages of concrete for individual beam segments during construction and the stress activations were not neglected, which is very important for this type of bridge construction.

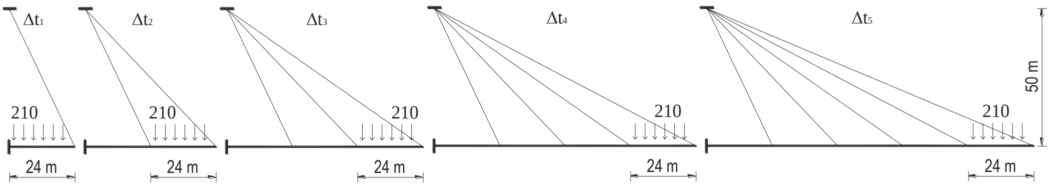

Figure 3. *Static systems and loads for considered time intervals*

The discretization of the total time was carried out in accordance with Figure 4. The total time is divided into 12 intervals following the adopted technology of construction of the main structure of the bridge. The calculation does not neglect different ages of concrete at the time of loading, ie the construction of the next beam (slab) segment includes the load from that segment, and it consideres the contribution of concrete rheology in previously constructed concrete segments according to the time scale.

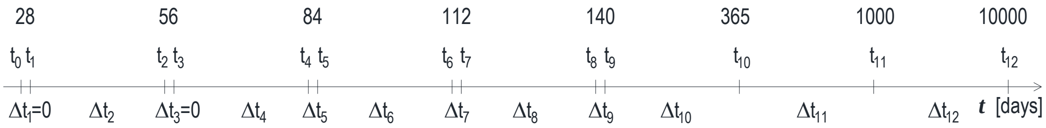

Figure 4. *Time discretization in accordance with the planned construction of the bridge*

2.1. ANALYSIS OF CALCULATION RESULTS

After the calculations, the characteristic stress diagrams in steel cables are given (Figures 5 and 6), as well as the diagrams of deflections (Figures 7 and 8) and bending moments (Figures 9 and 10) of the main concrete beam for the cases with and without cable tensioning.

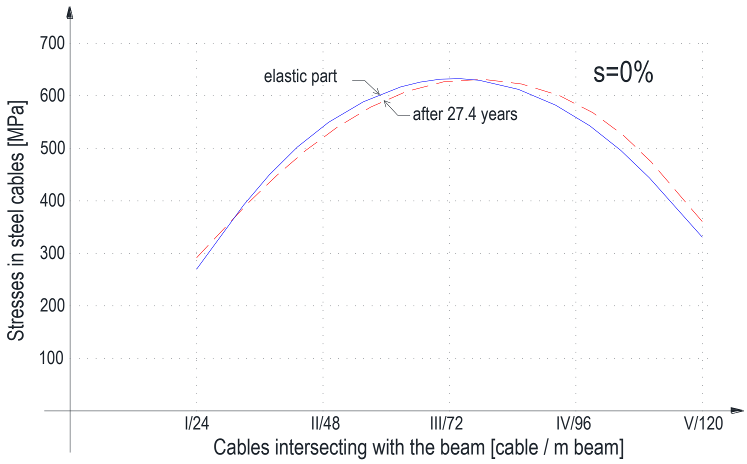

Figure 5. *Stresses in steel cables*

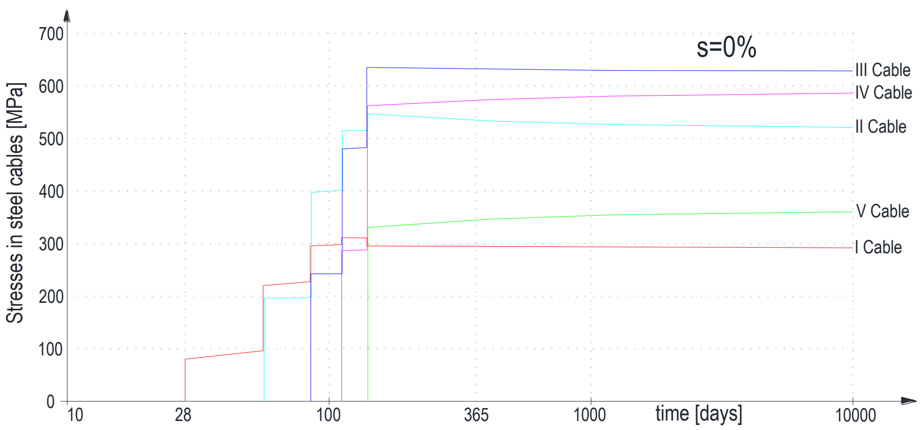

Figure 6. *Stress change in steel cables over time*

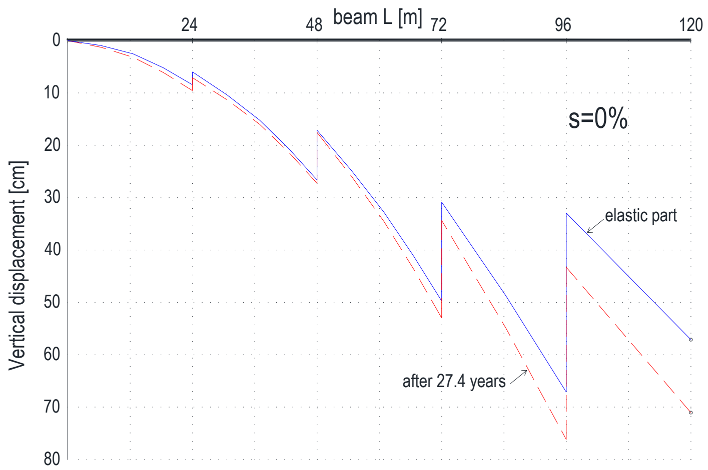

Figure 7. *Vertical displacement of concrete beam segments*

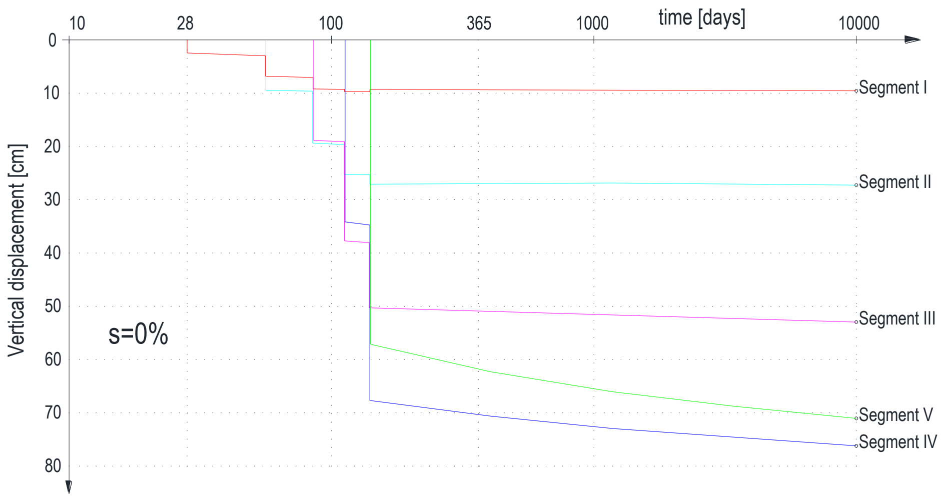

Figure 8. *Vertical displacement of concrete beam segments in time*

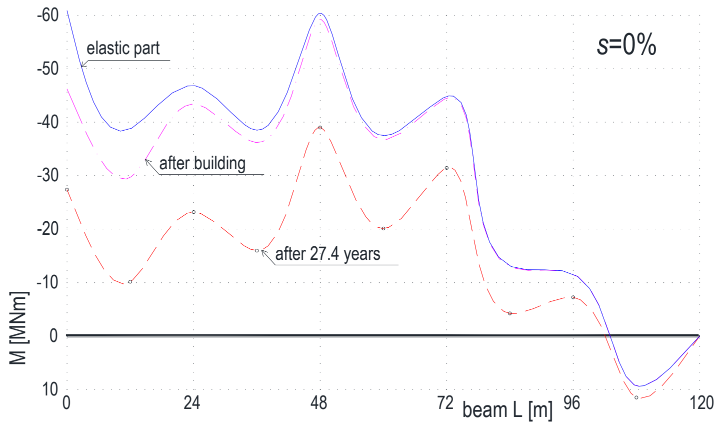

Figure 9. *Bending moments along the beam (for s=0%)*

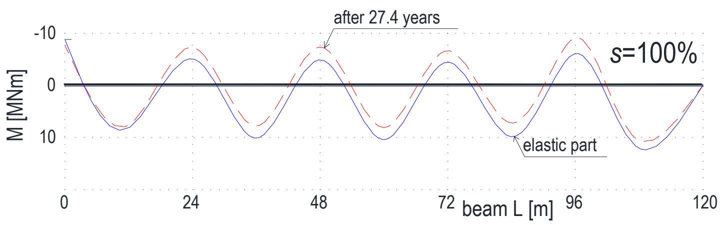

Figure 10. *Bending moments along the beam (for s=100%)*

Although some of the assumptions introduced in this example do not apply for all common cases of similar bridges in practice, significant impacts and their change in characteristic moments can be seen through successive design analyses of the bridge structure, with the emphasis on the main concrete beam, thus, through these analyses, useful conclusions can be drawn, when it comes to the contribution of the viscous properties of concrete in cable-stayed bridge structures [5]:

- Tightening of cables during segmental construction of the bridge, to continuously maintain the horizontality of the span beam, ie the projected elevation of this beam, has a beneficial effect, because it maintains the desired level of the main beam, and thus reduces the sensitivity of concrete to shrinkage and creep. This condition is achieved in the case of fixed suspension points (cable stiffness s=100%) in the calculation, which means that the diagram of bending moments along the main beam is fairly uniform and has a shape typical for a continuous beam (Figure 10). Due to the rheology of the concrete, this diagram moves slightly upwards over time, thus the beam in the middle area of the span is somewhat unloaded, and in the support zones it is additionally stressed by the approximately same absolute value of the moment.
- In the case of actual numerical stiffness of cables (s=0%), where, in fact, during the construction of each new segment of the main beam, the cables of previously constructed segments are not tightened, that is, beam deflections depend on the real stiffness of cables and on the load in each calculation interval (they grow), the creep and shrinkage of the concrete cause significant changes in the stress of the main structure (beam) over time. In this case, the bending moments of the main beam decrease over time, and the deflections increase. However, it should be noted that the initial bending moments are quite unfavorable for this type of bridge construction. Namely, these bending moments, unlike in the previously analyzed state (s = 100%), have a high intensity and are negative along the entire length of the beam, except in the last segment where the positive values in absolute are significantly lower than the negative moments.
- In general, the rheology of concrete causes a more favorable distribution of the bending moments in the main beam and reduces the amplitudes of the moments over time for this type of structure and the construction technology. This relative change (reduction) of moments, compared to the initial values, is much more pronounced in the case when the cables are not tightened during construction (s=0%), while in the case of tightening the cables (s=100%) the initial moments are much smaller, and they change very little over time, so the contribution of concrete rheology, in this case, can be neglected.

3. CONCLUDING REMARKS

In general, based on the analyzed example and remarks, it can be concluded that the presented calculation model, within the assumed assumptions, can be used for analysis of different cases in engineering practice, when it comes to controlling serviceability limits with the considerration of time deformations of concrete through appropriate layered elements of structure. The generalized calculation model also includes statically indeterminate, where the impacts change in the cross-sections without changing the external load, which is a great contribution of this calculation model. It is also important to point out that more complex cases can be analyzed in practice, such as e.g. subsequent interventions due to the strengthening and rehabilitation of existing structural systems, and various construction methods (cantilever systems, continuation of prefabricated elements with additional concrete or prestressing cables, etc).

The conclusion is that in the calculation of most structures it is necessary to include viscoelastic properties of materials, primarily shrinkage and creep of concrete, because stresses and deformations can change significantly over time [6]. Pressed concrete layers are in principle unloaded, and additional impacts are taken over by steel elements (reinforcement). It is also very important that these impacts are observed in the real conditions, during the applied construction technology through all characteristic time intervals. Considering the analysis of the serviceability limit states, any redistribution of impacts and changes (increment) in deflection should be reduced to an acceptable measure, thus possible negative effects in the building operation should be prevented.

The example of the suspension bridge analyzed herein indicates the need to tighten the cables of the previously built concrete segment of the main beam when adding each new segment. This achieves uniformity of the stresses and deflections along the entire length of the main beam, and the impact of concrete rheology is reduced to a minimum. Otherwise, if the cables are not tightened, the stresses and deflections along the beam are notably uneven, with the rheology of concrete in these conditions reducing the ultimate stresses and increasing deflections.

LITERATURE

[1] A. Cumbo and R. Folić, "Layered Finite Elements in the Analysis of Composite Structures Exposed to Long-Term Effect," *Građevinar,* vol. 69, no. 11, pp. 991-1005, 2017.

[2] R. Folić and A. Cumbo, "Theoretic Bases for Analysis of Viscous Deformations in Composite Beams Using the Finite Elements Method," *Bulletins for Applied & Computing Mathematics,* Vols. BAM-1770, pp. 17-28, 2000.

[3] M. A. Chiorino and D. J. Carreira, "Factors Affecting Creep and Shrinkage of Hardened Concrete and Guide for Modelling," *ICJ The Indian Concrete Journal,* vol. 86, no. 12, pp. 11-24, 2012.

[4] R. Folić, V. Radonjanin and M. Malešev, "Design and Analysis of Steel-Concrete Composite Structure," in *Introductinary - Invited paper on 6th Greek National Conference on Metal Structures*, Athens, 2008. pp. 72 – 87.

[5] M. Sassone and C. Casalegno, "Evaluation of the Structural Response to the Time-Dependent Behaviour of Concrete: Part 2 - A General Computational Approach," *ICJ The Indian Concrete Journal,* vol. 86, no. 12, pp. 39-51, 2012..

[6] R. C. Lagos, M. C. Kupfer, S. T. Sanhueza and F. V. Cordero, "Costanera Center - Shortenings Due to Elastic Deformation, Creep and Shrinkage of Concrete in a 300-m Tall Building," *ICL The Indian Concrete Journal,* vol. 86, no. 12, pp. 53-60, 2012.