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ANALIZA LINIJSKOG KOEFICIJENTA PROLAZA TOPLOTE JEDNOSTAVNOG BALKONSKOG TOPLOTNOG MOSTA

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Apstrakt:

U ovom radu je analiziran linijski koeficijent prolaza toplote jednostavnog toplotnog mosta koji se javlja na spoju uniformne balkonske ploče i zida. Model zavisi od pet parametara: debljina, dužina i toplotna provodljivost balkonske ploče; debljina i toplotna provodljivost zida. Rezultati istraživanja ukazuju da dužina balkonske ploče ne utiče na linijski koeficijent prolaza toplote, dok je zavisnost od druga dva parametra ploče linearnog tipa. S druge strane, zavisnost od parametara zida je komplikovanijeg oblika, a odgovarajući izrazi koji opisuju ovu zavisnost su uspješno izvedeni. Ovo istraživanje predstavlja prvi korak u pronalaženju semi-empirijske jednačine, koja bi mogla uspješno opisati linijski koeficijent prolaza toplote toplotnog mosta koji se javlja na spoju betonske ploče i homogenog višeslojnog zida.

Ključne riječi: prenos toplote, toplotni most, balkon, numerička simulacija

ANALYSIS OF LINEAR THERMAL TRANSMITTANCE OF A SIMPLE BALCONY THERMAL BRIDGE

Abstract:

The linear thermal transmittance of a simple balcony thermal bridge with a uniform balcony slab and wall was analysed. In a five-parameter model (thickness, length and thermal conductivity of a balcony slab; thickness and thermal conductivity of an adjacent wall), it was found that the length of the balcony slab does not influence the thermal transmittance, while its dependence on the other two slab parameters is linear. The influence of wall parameters on the linear transmittance proved to be more complicated, but appropriate expressions were successfully found. The study presents the first step in the search of a semi-empirical formula that can successfully describe the linear transmittance of a balcony thermal bridge with a homogeneous multi- layer wall.

Keywords: heat transfer, thermal bridge, balcony, numerical simulation

1. INTRODUCTION

Due to ecological and also economy reasons, energy efficiency of buildings is gaining on importance. Laws regarding energy efficiency are getting stricter and we are past the stage, when heat losses can be simply treated by just adding a thicker insulation layer and/or better fenestration. Heat losses and gains in buildings have to be understood on a deeper level, especially including the heat losses through thermal bridges and ventilation. Some case studies show, that the heat losses through thermal bridges can reach 50% of all building heat losses or, in some special cases, even larger ratio is possible [1]. Specifically, balcony thermal bridges and thermal bridges around fenestration are the most critical.

Nevertheless, the proper treatment of the heat losses through thermal bridges, at least in Slovenia, is usually inaccurate. For example, [2] allows treating heat losses through thermal bridges just by adding general value of 0.06 W/m^2K to the heat flow rate per square meter if the linear transmittance of thermal bridges does not exceed 0,2 W/mK. Nevertheless, this general value is used even in cases where thermal bridges with linear transmittance larger than 0,2 W/mK are present. There are probably several reasons for this practice. One of these reasons is, that the calculation of the linear transmittance of thermal bridges is more complicated than simple calculation of the heat loss trough uniform wall. These calculations usually require usage of expensive software, or consume time and require good knowledge of numerical calculation tools.

Apart from numerical calculations, SIST EN ISO 14683 [3] in certain cases allows simpler treatment of thermal bridges. One of the techniques is to use cataloged values. Nevertheless, values of linear transmittance in catalog in [3] are usually rater large in comparison with real values [1] and therefore, their usage is disfavored. Another method would be to use semi-empirical expressions, similar to those used in treatment of heat losses through ground, defined in [4].

Such expressions for some thermal bridges are for example proposed by [5, 6]. Nevertheless, in these studies, the expression for linear thermal transmittance of thermal bridges was assumed to be of the form

$$\psi = l_{tb} U_{tb} + \frac{f(r_w)}{\Delta\theta} \tag{1}$$

where:

ψ is the linear thermal transmittance of the thermal bridge $[W/mK]$,

l_{tb} is the thickness of the thermal bridge (e. g. balcony slab thickness) $[m]$,

U_{tb} is the thermal transmittance of the thermal bridge (e. g. balcony slab) $[W/m^2K]$,

$f(r_w)$ is the linear function of thermal resistances of adjacent walls $[W/m]$,

$\Delta\theta$ is the temperature difference between interior and exterior $[^oC]$.

For the $f(r_w)$ linear regression model is proposed. Although the above expression is logical, real situation is probably richer in structure. This is, for example, indicated by cataloged values in [3], where it can be noted, that the value of linear transmittance can

strongly depend on the position of insulation layer. Likewise, the values of $f(r_w)$ in [5, 6] show stochastic features, although no stochastic nature is expected.
Therefore, in this paper, a semi-empirical expression for a linear thermal transmittance of a simple balcony thermal bridge will be proposed without any assumption on the form. Analysis will be done on a simple model case of a balcony consisting of a uniform single layer balcony slab and uniform single layer wall that can be later used as a base for more complicated analysis of a thermal bridge with a multiple layer wall.
The paper is structured as following. Theoretical introduction is given is section 2. In section 2.1, basic expressions are defined and method of calculation of a linear thermal transmittance of a thermal bridge is explained. In section 2.2, software used in calculations is introduced. Results are given in sections 3 and 4 In section 3, single parameter variations are presented. In section 4, semi-empirical expression for a linear thermal transmittance of a balcony thermal bridge is given. Conclusions are stated in section 5.

2. FRAMEWORK

Analysis is done on a simple model case of a balcony, consisting of a uniform single layer balcony slab and uniform single layer wall (see Fig. 1), later to be extended on a multiple layer balcony slab and wall. This simple example therefore depends on five parameters: thermal conductivity of the wall (λ_w), thermal conductivity of a balcony slab (λ_s), thickness of a wall (d_w), thickness of a balcony slab (d_s) and length of a balcony slab (l). Parameters l_1, l_2 and l_3 were set to 2.5 m.

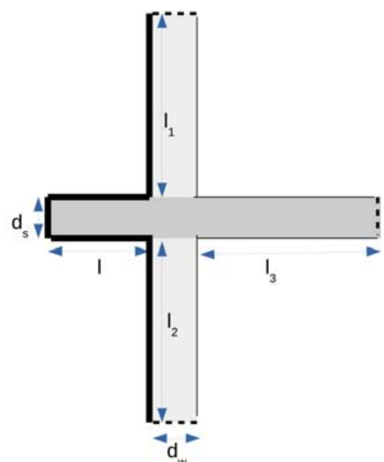

Figure 1. Two dimensional model of a simple balcony thermal bridge

2.1 BASIC DEFINITIONS

The overall heat loss of a building can be described by a heat transfer coefficient H, defined as a heat flow rate trough building's thermal envelope, divided by temperature difference between inside and outside of a building

$$H = \frac{\Phi}{\Delta\theta} \tag{2}$$

where:

H is a heat transfer coefficient $[W/K]$,

Φ is a heat flow rate $[W]$,

$\Delta\theta$ is the temperate difference between inside and outside of the building $[^{o}C]$.

The heat transfer coefficient depends on dimensions of the building. Larger is the surface of thermal envelope of the building, higher, in general, is the heat transfer coefficient. On the other hand, heat loss trough a specific wall is specified by thermal transmittance: a heat flow rate through a unit area of wall, divided by temperature difference on both sides of a wall

$$U = \frac{\Phi}{A\Delta\theta} \tag{3}$$

where:

U is a thermal transmittance $[W/m^2K]$,

Φ is a heat flow rate $[W]$,

A is an area of the wall $[m^2]$,

$\Delta\theta$ is a difference in air temperatures on both sides of the wall $[^{o}C]$.

It is expected than, that the heat transfer coefficient can be simply calculated by summing up thermal transmittances through all thermal envelope components multiplied by their area. Nevertheless, this is not a case, as heat loss trough thermal bridges, like wall junctions, balconies, around windows and similar geometrical and structural object, is more complicated. Namely, heat flow rate in these cases in not perpendicular to the wall surface as in the case of homogeneous building components. Therefore, more precisely, the relation between thermal transmittances and heat transfer coefficient, can be written as

$$H = \sum_i U_i A_i + \sum_j \psi_j l_j + \sum_k \chi_k \tag{4}$$

where:
H is a heat transfer coefficient $[W/K]$,
U_i is a thermal transmittance of the i-th wall $[W/m^2K]$,
A_i is an area of the i-th wall $[m^2]$,
ψ_j is a linear thermal transmittance of the j-th linear thermal bridge $[W/mK]$,
l_j is a length of the j-th linear thermal bridge $[m]$,
χ_k is a point thermal transmittance of k-th point thermal bridge $[W/K]$.

So, according to their geometry, thermal bridges can be divided into linear and point thermal bridges. Linear thermal bridges can be presented by line (e. g. junction of two walls, thermal bridge around windows…), while point thermal bridges can be presented by a point (e. g. junction of three walls, junction of a pillar with a roof…). According to [3], in most cases, point thermal bridges can be neglected.

Heat flow rate trough linear thermal bridge is calculated by solving a Laplace equation

$$\nabla^2\theta = 0 \tag{5}$$

where:
θ is a temperature in a given point $[^oC]$,
on a two dimensional geometrical model of a thermal bridge with its surroundings. An example of such geometrical model for a balcony thermal bridge is presented in Fig. 1. Details of how to construct two dimensional model of a thermal bridge and what boundary conditions have to be used, is defined in [7].
The following boundary conditions are used:

- Density of heat flow rate perpendicular to the cut-off surfaces (dotted line in Fig 1.) has to be zero (adiabatic boundary condition)

$$q = \lambda \frac{\partial\theta}{\partial r} = 0 \tag{6}$$

where:
q is density of the heat flow rate perpendicular to cut-off surfaces $[W/m^2]$,
λ is thermal conductivity $[W/mK]$,
$\frac{\partial\theta}{\partial r}$ is partial derivative of a temperature field in a direction perpendicular to cut-off surface $[K/m]$.

- Density of heat flow rate perpendicular to surfaces facing exterior (thick full line in Fig. 1), has to be

$$q = \frac{\theta_e - \theta_{es}}{R_{se}} = 0 \tag{7}$$

where:

q is density of the heat flow rate perpendicular to the surfaces facing exterior $[W/m^2]$,

θ_e is external temperature $[^oC]$,

θ_{es} is temperature on a boundary (surfaces facing exterior) $[^oC]$,

R_{se} is external surface resistance $[m^2K/W]$.

- Density of heat flow rate perpendicular to surfaces facing interior (thin full line in Fig. 1), has to be

$$q = \frac{\theta_i - \theta_{is}}{R_{si}} = 0 \tag{8}$$

where:

q is density of the heat flow rate perpendicular to the surfaces facing interior $[W/m^2]$,

θ_i is internal temperature $[^oC]$,

θ_{is} is temperature on a boundary (surfaces facing interior) $[^oC]$,

R_{si} is internal surface resistance $[m^2K/W]$.

Thermal coupling coefficient (L_{2D}) is than defined as an integral of a density of a heat flow rate over outside or over inside surfaces, divided by the external and internal temperature difference:

$$L_{2D} = \frac{\int q dr}{\Delta\theta} \tag{9}$$

where,

L_{2D} is thermal coupling coefficient $[W/mK]$,

q is density of a heat flow rate $[W/m^2]$,

$\Delta\theta$ is the difference between external and internal temperatures $\left[{}^{o}C\right]$.

From thermal coupling coefficient, the linear thermal transmittance is them simply calculated as

$$\psi = L_{2D} - LU_{w} \tag{10}$$

where:

ψ is linear thermal transmittance $\left[W/mK\right]$,

L_{2D} is thermal coupling coefficient $\left[W/mK\right]$,

$L = l_1 + l_2 + d_s$ is the length of the modeled element $\left[m\right]$,

U_w is the thermal transmittance of the wall $\left[W/m^2K\right]$.

Here, the external dimension of the wall ($L = l_1 + l_2 + d_s$) is used. Alternatively, one can also use internal wall dimensions ($l_1 + l_2$) as long as internal dimensions are used consistently when calculating heat transfer coefficient (see Eq. (2)). In this study, external dimensions are used, as its usage is prescribed in [2].

2. 2. FREE FEM PDE SOLVER

A thermal coupling coefficient of a balcony was calculated using Free Fem [8]. Free Fem is an open source partial differential equation solver based on a final element method. It is capable of solving 2D and 3D partial differential equation and gives a user a good control over a calculation details. User writes a short script in which he or she defines geometry, type of final elements, method of how the final elements are spanned (e. g. adapt mash), differential equation to be solved, initial and boundary conditions, details of calculations (e. g. precision, numerical method for dealing with matrices), etc.. As all information are defined trough script, Free Fem is more tedious to use than other graphically based PDE solvers, but in return gives better control over input parameters and greater freedom in data analysis. In this study, uniform triangle mash was used with around 25 000 P2 parabolic elements [8] and the precision of calculation was set to a relative error of 10E-9.

3. SINGLE PARAMETER VARIATIONS

Before overall fit of all parameters is done, linear thermal transmittance of the balcony will be calculated by varying one of the parameters, while all other parameters are kept fixed. By this method, one can gain some insight into importance and influence of a specific parameter on a linear thermal transmittance of a balcony. To be able to get a better understanding of the shape of the function describing the dependence of linear thermal transmittance of a balcony on a specific parameter, parameters were also varied outside their realistic values. The results are shown in Figs. 2-4.

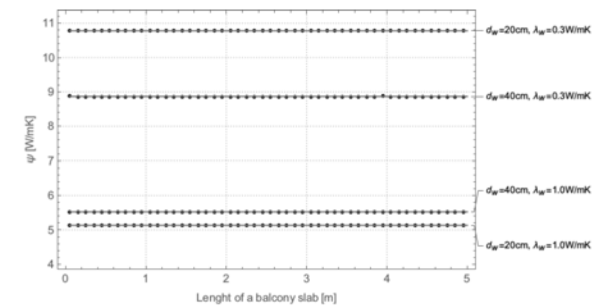

Figure 2. Influence of the length of a balcony slab on a linear thermal transmittance of the balcony

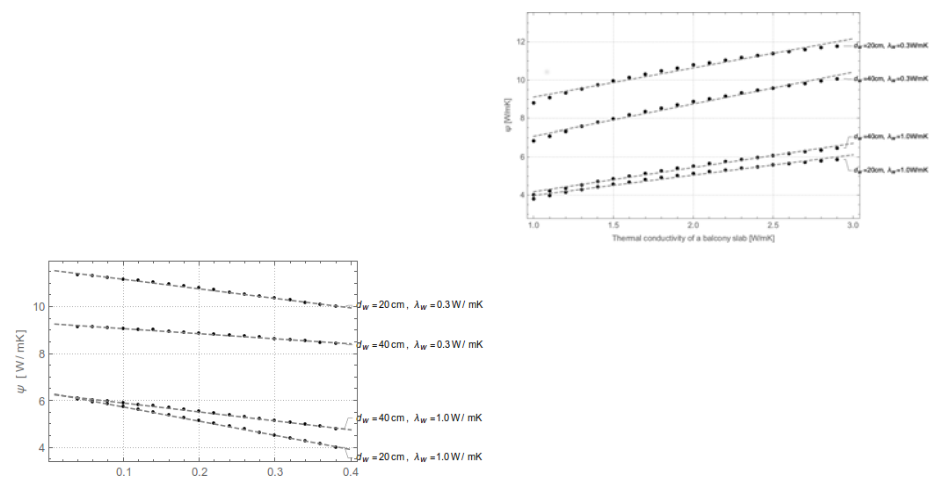

Figure 3. Influence of the thickness and thermal conductivity of a balcony slab on linear thermal transmittance of the balcony

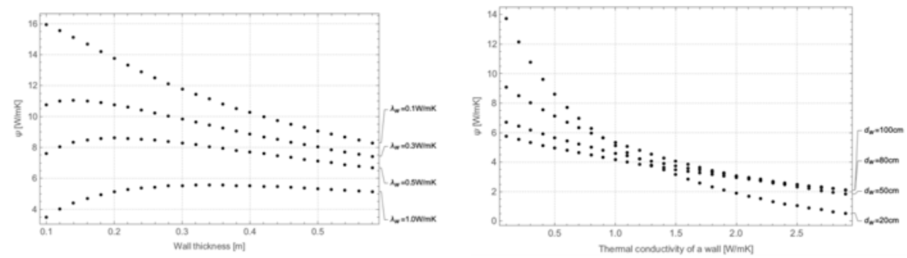

Figure 4. Influence of the thickness and thermal conductivity of a wall on linear thermal transmittance of the balcony

From Fig. 2, one can easily conclude, that the length of the balcony slab has no influence on thermal transmittance of the balcony. This is welcome fact, as it deduces the number of model parameters from five to four. It is also in accordance with the assumption made by [5, 6] and stated in Eq. (1). Furthermore, it can be deduced from Fig. 3 that the linear thermal transmittance of the balcony can be presented as a linear function of both thickness and the thermal conductivity of the balcony slab, which further simplifies the analysis. Surprisingly, it can be also perceived, that the linear thermal transmittance of the balcony falls with the increasing thickness of the balcony slab, which is unexpected and in contradiction to the assumption made by [5, 6]. Finally, from Fig. 4, it can be observed that the dependence of thermal transmittance of a balcony of thickness and conductivity of a wall is rich in structure, and more complicated regression function instead of linear should be used. From the above observations, the following solution is proposed: first, two different balcony slab thickness (d_{s1} , d_{s2}) and balcony slab thermal conductivities (λ_{s1} , λ_{s2}) will be chosen. Next, in section 4, at each combination of these fixed values, the function of thickens and the thermal conductivity of the wall that best describes the linear thermal transmittance of the balcony, will be found. From these data, one should than be able to deduce linear thermal transmittance of the balcony at any set of parameters by simple interpolation.

From Fig. 3 (left) it can be observed, that the dependence of linear thermal transmittance of the balcony on balcony slab thickness is strongly linear, so any two balcony slab thicknesses can be chosen. To be near the realistic span of parameters, d_{s1}=10cm and d_{s2} =20cm would be used, but any other similar choice would be equally good. On the other hand, the linearity of liner thermal transmittance of a balcony on balcony slab thermal conductivity is not so strong (Fig. 3, right), so representative parameters should be chosen more carefully. As values of linear thermal transmitance of in points λ_{s1} =1.5W/mK and λ_{s2} =2.5W/mK coincide with the linear function found by regression, these two values will be taken as reference values.

4. REGRESSION MODEL OF LINEAR THERMAL TRANSMITTANCE OF THE BALCONY

First, the dependence of linear thermal transmittance of the balcony on thermal conductivity of the wall will be studied. Namely, from Fig. 4, it can be seen that this dependency is easier to deduce. The fit was done using exponential function, arc-tan function and rational function. All three expressions fit data with a very good precision, although arc-tan function and rational function fit data somewhat better then exponential function. Due to simplicity, rational functions are used. In Fig 5 therefore, the dots present calculated values of linear thermal transmittance at fixed parameter values, while full line presents best rational function found by the regression of the form:

$$\psi = A(d_w) + \frac{B(d_w)}{C(d_w) + \lambda_w} \tag{11}$$

where:

ψ is a linear thermal transmittance of a balcony $[W / mK]$,

λ_w is a thermal conductivity of a wall $[W / mK]$,

$A(d_w), B(d_w), C(d_w)$ are functions of wall thickness.

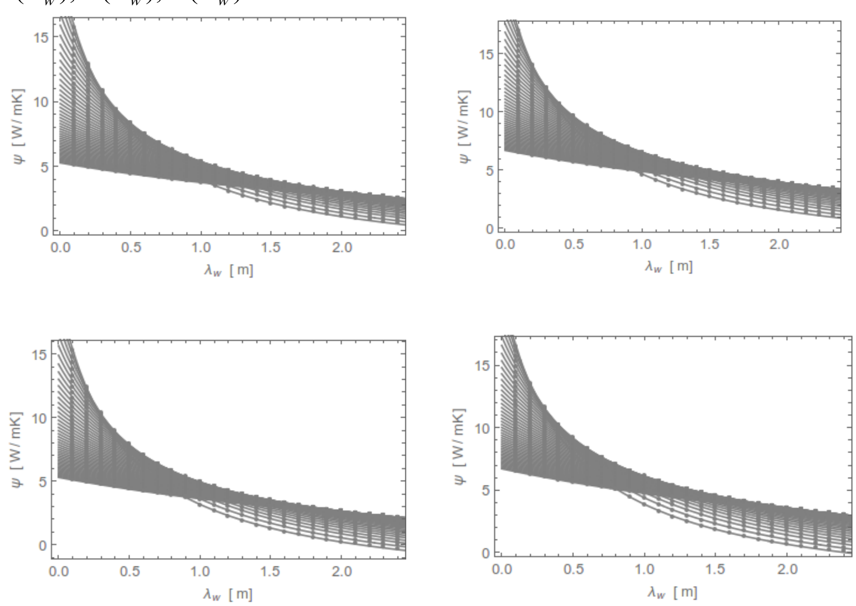

Figure 5. Dependence of the linear thermal transmittance of the balcony on the thermal conductivity of the wall (first row d_s1=10 cm, second row, d_s2=20 cm, first column λ_s1 =1.5 W/mK, second column λ_s2 =2.5 W/mK)

Multiple lines present variations of the wall thickness, while four diagrams present solutions at four sets of balcony slab thickness and thermal conductivity.

Regression functions $A(d_w)$, $B(d_w)$ and $C(d_w)$ are presented in Fig. 6-8. Dots present values of parameters belonging to different lines in Fig. 5, while full lines presents functions found by regression model. Similar as before, four diagrams present solutions at four sets of balcony slab thickness and thermal conductivity.

It is can be concluded, that the function $C(d_w)$ can be simply approximated with the linear function. The shapes of the functions $A(d_w)$ and $B(d_w)$ were harder to deduce, but it was fund out that the following expressions approximate the data with relatively good precision:

$$A(d_w) = \frac{1}{A_1 d_w + A_2} + A_3 + A_4 d_w + A_5 d_w^2, \tag{12}$$

$$B(d_w) = B_1 \ln(B_2 d_w + 1) + B_3 d_w, \tag{13}$$

$$C(d_w) = C_1 d_w, \tag{14}$$

where:

d_w is the balcony wall thickness $[m]$.

Coefficients A_i, B_i and C_i are given in Table 1.

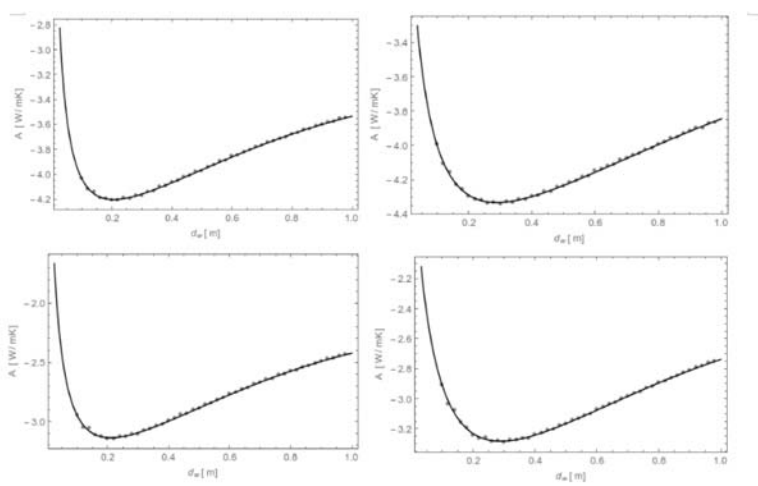

Figure 6. Function A(d_w) for different combinations of balcony slab thickenss and thermal conductivity (first row d_s1=10 cm, second row, d_s2=20 cm, first column λ_s1 =1.5 W/mK, second column λ_s2 =2.5 W/mK)

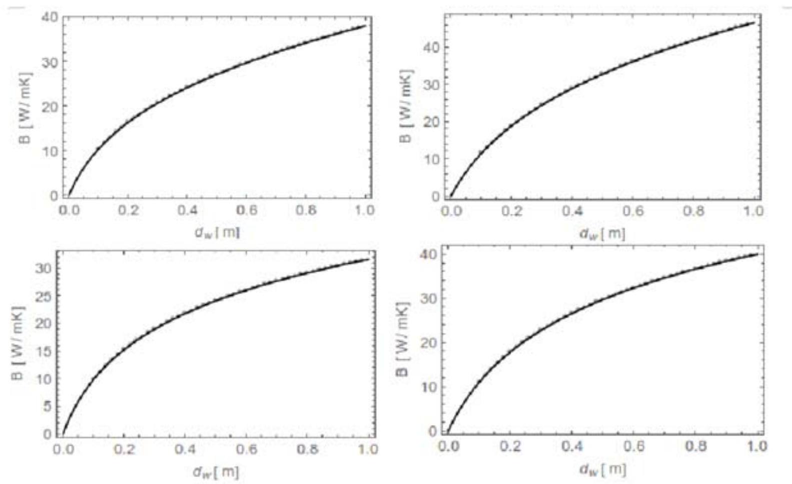

Figure 7. Function B(d_w) for different combinations of balcony slab thickenss and thermal conductivity (first row d_s1=10 cm, second row, d_s2=20 cm, first column λ_s1 =1.5 W/mK, second column λ_s2 =2.5 W/mK)

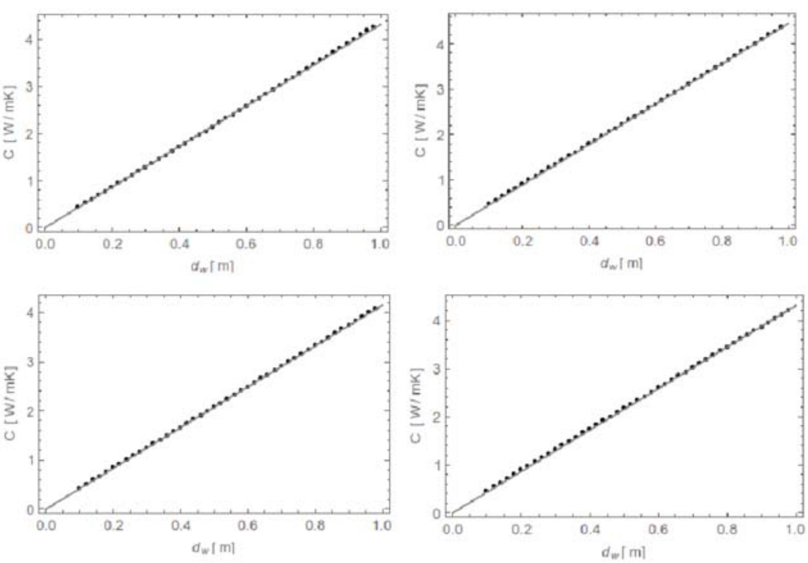

Figure 8. Function C(d_w) for different combinations of balcony slab thickenss and thermal conductivity (first row d_s1=10 cm, second row, d_s2=20 cm, first column λ_s1 =1.5 W/mK, second column λ_s2 =2.5 W/mK)

The linear thermal transmittance of the balcony with parameters $d_w, d_s, \lambda_{w,} \lambda_s$ can now be calculated in a following way:

- First one calculates the linear thermal transmittance at given wall thickness and thermal conductivity at all four combinations of representative balcony slab thicknesses (d_{s1} =10cm and d_{s2} =20cm) and thermal conductivities (λ_{s1} =1.5W/mK and λ_{s2} =2.5W/mK): $\psi_{11}(d_w, \lambda_w, d_{s1,} \lambda_{s1})$, $\psi_{12}(d_w, \lambda_w, d_{s2,} \lambda_{s1})$, $\psi_{21}(d_w, \lambda_w, d_{s1,} \lambda_{s2})$ and $\psi_{22}(d_w, \lambda_w, d_{s2,} \lambda_{s2})$, using Eq. (11)-(14) and coefficients from Table 1.
- From above values and using interpolation, the linear thermal transmittance at wanted balcony slab thickness and thermal conductivity is calculated

$$\psi_1(d_s, \lambda_{s1}) = \frac{\psi_{12}(d_{s2,} \lambda_{s1}) - \psi_{11}(d_{s1,} \lambda_{s1})}{d_{s2} - d_{s1}}(d_s - d_{s1}) + \psi_{11}(d_{s1,} \lambda_{s1}) \quad (15)$$

$$\psi_2(d_{s,} \lambda_{s2}) = \frac{\psi_{22}(d_{s2,} \lambda_{s2}) - \psi_{21}(d_{s1,} \lambda_{s2})}{d_{s2} - d_{s1}}(d_s - d_{s1}) + \psi_{21}(d_{s1,} \lambda_{s2}) \quad (16)$$

$$\psi(d_{s,} \lambda_s) = \frac{\psi_2(d_{s,} \lambda_{s2}) - \psi_1(d_{s,} \lambda_{s1})}{\lambda_{s2} - \lambda_{s1}}(\lambda_s - \lambda_{s1}) + \psi_1(d_{s,} \lambda_{s1}) \quad (17)$$

Table 1. The coefficients for the regression functions $A(d_w)$, $B(d_w)$ *and* $C(d_w)$

	$\psi_{11}(d_{s1,} \lambda_{s1})$ $[W/mK]$	$\psi_{12}(d_{s2,} \lambda_{s1})$ $[W/mK]$	$\psi_{21}(d_{s1,} \lambda_{s2})$ $[W/mK]$	$\psi_{22}(d_{s2,} \lambda_{s2})$ $[W/mK]$
$A_1\ [K/W]$	11.6	9.55	5.9	3.88
$A_2\ [mK/W]$	0.17	0.19	0.3	0.29
$A_3\ [W/mK]$	-4.99	-4.05	-5.29	-4.61
$A_4\ [W/m^2K]$	2.06	2.31	1.72	2.36
$A_5\ [W/m^3K]$	-0.69	-0.79	-0.43	-0.73
$B_1\ [(W/mK)^2]$	12	12.1	18.6	18.93
$B_2\ [1/m]$	12.3	12.62	7.88	8.03
$B_3\ [W^2/m^3K^2]$	6.83	-0.03	5.85	-1.69
$C_1\ [W/m^2K]$	4.31	4.14	4.45	4.31

5. CONCLUSION

A linear thermal transmittance of a simple balcony model with uniform single layer slab and uniform single layer wall was studied. It was found that the length of balcony slab does not have influence on the linear thermal transmittance, while it linearly depends on two other slab parameters: thickness and thermal conductivity. The influence of the wall parameters on a linear transmittance proved to be more complicated, but appropriate expressions were successfully found. Finally, a method that gives the linear thermal transmittance of a simple balcony thermal bridge for any combination of all five parameters to a good precision, is presented. The study presents the first step in a search for a semi-empirical formula that can describe the linear thermal transmittance of a balcony with multiple layer homogeneous walls. If successful, the same method can be used for deducing semi-empirical expressions for other types of critical thermal bridges (like, for example, thermal bridge around windows).

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