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THE CENTRE OF CIRCLE DETERMINATION BY GEODETIC MEASUREMENTS

Abstract

The problem of circle's centre determination by using geodetic data is related with influences of inevitable errors as a result of measurement process as well as with the issue of model's influence itself. In this research the experiment was conducted with aim to find out the influence of measurements error and the influence of model for centre of circle determination. The two centres of two different centres were determined by direct measurements and with the model based on the points belonging to the circle. The difference between centres determined by direct measurements and circles' centres determined from the model is actually the models' influence. Obtained results showed high concordance between the centeres determined both with measurements and model.

Keywords: accuracy, precision, statistical hypothesis testing

ОДРЕЂИВАЊЕ ЦЕНТРА КРУГА ГЕОДЕТСКИМ МЕТОДАМА

Сажетак

Одређивање центра круга геодетским методама повезано је са утицајем неизбјежних грешака у процесу мјерења као и са утицајем самог модела одређивања центра.У овом истраживању спроведен је експеримент са циљем да се одреди утицај грешке модела и утицај грешака мјерења на одређивање центра круга примјеном геодетских метода мјерења. Разлика између центра круга одређеног на основу директних мјерења и центра круга одређена на основу модела указује на утицај модела. Добијени резултати указују на високу сагласност између центара добијених директним мјерењем и центара добијених на основу модела.

Кључне ријечи: тачност, прецизност, тестирање статистичких хипотеза

1. INTRODUCTION

The circle and its center determination are often issue in mechanical engineering. Mechanical elements are usually made on the machine lathe, and it is considered that elements produced in this manner are of quite accurate dimensions. However, because of different influences every element is produced with some deviations. If the dimensions are inside predefined tolerances, they are considered to satisfy required level of quality. Otherwise, the elements must be remedied or repaired. The control of produced mechanical element's geometry is the issue of geodetic domain mostly caused by the increasing accuracy and efficiency of contemporary geodetic technologies. Also, the model of least squares for data processing commonly used in geodesy is appropriate for center and radiuses of circle determination.

In most cases when the circle's center and radiuses are required, measurements provided by mechanical engineering equipment either do not provide possibility for center determination or are very expensive. Geodetic technologies and methods provide possibility for "ideal" center and radiuses determination on the base of the measured points around the circle on the high level of accuracy and efficiency.

The geodetic methods for determining circle's center and radiuses were utilized and proven in practice where the high accuracy was required [1, 2]. Furthermore, it was proven that it is possible to obtain high accuracy of zenith angles when the lines of sights are short [3]. This research was provided to find out the possible accuracy of circle's center determination from the points located on the circle's line by comparison with the circle's center determined on the base of measurements.

2. MATERIALS AND METHODS

The data (coordinates of points) for circle's center determination were obtained by measurements which were adjusted by the means of least squares [4]. There were measured points located on two circles. Each circle was approximated with twelve points and their centers were marked also. The data are given in table 1.

Table 1. The coordinates of points on the circles and their centers determined by measurements[m]

Point N°	y (1st circle)	x (1st circle)	y (2nd circle)	x (2nd circle)
0	10.3155	10.0000	10.9239	9.9660
1	10.3595	9.9880	10.9499	9.9592
2	10.3913	9.9558	10.9689	9.9404
3	10.4028	9.9118	10.9759	9.9146
4	10.3910	9.8678	10.9692	9.8887
5	10.3586	9.8359	10.9502	9.8697
6	10.3147	9.8241	10.9243	9.8625
7	10.2711	9.8361	10.8985	9.8693
8	10.2389	9.8685	10.8793	9.8882
9	10.2273	9.9127	10.8722	9.9141
10	10.2393	9.9566	10.8791	9.9400
11	10.2716	9.9887	10.8981	9.9590
12 (y_c^0, x_c^0)	10.31511	9.91217	10.92411	9.91430

The model for centers determination is as follows:

$$R_i = \sqrt{(y_i - y_c)^2 + (x_i - x_c)^2} \tag{1}$$

where:

- R_i – radius,
- y_i - y coordinate of ith point,
- x_i - y coordinate of ith point,
- y_c - y coordinate of circle's center (unknown) and
- x_c - x coordinate of circle's center (unknown).

Linearization of equation (1) leads to:

$$R_i + v_i = R_0 + \frac{\partial R}{\partial y_c}|_{y_c=y_c^0}\Delta y_c + \frac{\partial R}{\partial x_c}|_{x_c=x_c^0}\Delta x_c \quad (2)$$

$$\frac{\partial R}{\partial y_c} = -\frac{y_i - y_c^0}{\sqrt{\left(y_i - y_c^0\right)^2 + \left(x_i - x_c^0\right)^2}} \quad (3)$$

$$\frac{\partial R}{\partial x_c} = -\frac{x_i - x_c^0}{\sqrt{\left(y_i - y_c^0\right)^2 + \left(x_i - x_c^0\right)^2}} \quad (4)$$

After forming the linear equation system, we obtain twelve equations and have only two unknowns. The solution is obtained by the means of least squares model:

$$[vv] = min \quad (5)$$

$$v = Ax + f \quad (6)$$

$$x = -(A^T A)^{-1} A^T f = -N^{-1} A^T f = -Q_x A^T f \quad (7)$$

where:

$$Q_x = \begin{bmatrix} q_{yy} & q_{yx} \\ q_{xy} & q_{xx} \end{bmatrix}$$

$$f = \begin{bmatrix} R_0 - R_1 \\ \dots \\ R_0 - R_{11} \end{bmatrix}$$

$$x = \begin{bmatrix} y_c \\ x_c \end{bmatrix} = \begin{bmatrix} y_c^0 + \Delta y_c \\ x_c^0 + \Delta x_c \end{bmatrix}$$

$$P = E = \begin{bmatrix} 1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 1 \end{bmatrix}$$

Q_x is cofactor matrix, f is vector of free terms, x is vector of unknown circle's center coordinates and P is matrix of weights. The approximate value of radius R_0 is obtained as follows:

$$R_{0_i} = \sqrt{(y_i - \bar{y})^2 + (x_i - \bar{x})^2} \quad (8)$$

$$\bar{y} = y_c^0 = \frac{1}{12}\sum_{i=0}^{11} y_i\,; \quad \bar{x} = x_c^0 = \frac{1}{12}\sum_{i=0}^{11} x_i$$

$$R_0 = \frac{1}{12}\sum_{i=0}^{11} R_{0_i}$$

The root mean square errors of obtained circle's center coordinates is calculated as follows:

$$m_{y_c} = m_0\sqrt{q_{yy}} \quad (9)$$

$$m_{x_c} = m_0\sqrt{q_{xx}} \quad (10)$$

Final values of circle's radiuses were obtained by following formula (1).

Statistical hypotheses about equality of circle's measured and determined center equality are tested by following student's test statistics:

$$t_y = \frac{\delta y}{m_{\delta y}} = \frac{y_c - y'_c}{\sqrt{m_{y_c}^2 + m_{y'_c}^2}} = t_{1-\alpha;f} \quad (11)$$

$$t_x = \frac{\delta y}{m_{\delta y}} = \frac{x_c - x'_c}{\sqrt{m_{x_c}^2 + m_{x'_c}^2}} = t_{1-\alpha;f} \tag{12}$$

where:

- t_y, t_x – test statistics for equality of circle's centers obtained from original measurements (adjusted measurement from geodetic network) and circle's centers obtained from points on the circle,
- y_c, x_c – circle's center coordinate determined from the points on the circle,
- y'_c, x'_c – circle's center's coordinate determined from the direct measurements,
- $m_{y_c}^2, m_{x_c}^2$ – mean square errors of the y_c, x_c coordinates, respectively,
- α – level of significance ($\alpha = 0.05$) and
- f – degrees of freedom.

3. RESULTS AND DISCUSSION

Geodetic measurements were provided in a room under artificial light. For the measurement the total station was utilized with following measuring uncertainties:

- $\sigma_p = 0.5''$ for directions,
- $\sigma_Z = 0.5''$ for zenith angles and
- $\sigma_Z = 2$ mm+2 ppm for slope distances in reflector less mode.

The conditions and process of measurements are illustrated by figure 1. Utilizing proposed model (equations 1-10) the results were obtained and given in table 2.

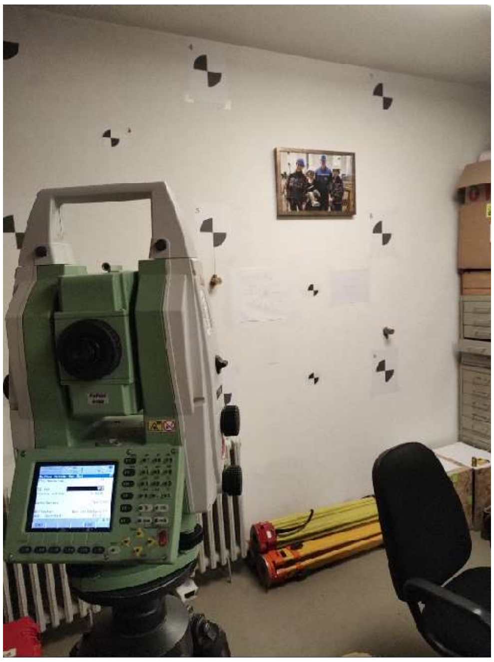

Figure 1. The conditions and total station during measurements

Table 2. The results of circles' centers coordinate determination.

Point N°	y (1st circle) [m]	m_y [mm]	x (1st circle) [m]	m_x [mm]	y (2nd circle) [m]	m_y [mm]	x (2nd circle) [m]	m_x [mm]
y_c^0, x_c^0	10.31513	-	9.91216	-	10.92413	-	9.91431	-
y_c, x_c	10.31506	0.05	9.91213	0.03	10.92410	0.04	9.91429	0.03
y'_c, x'_c	10.31511	0.02	9.91217	0.03	10.92411	0.01	9.91430	0.03

After utilization of statistical hypothesis test (formulas: 11 and 12) the obtained results are given in table 3.

Table 3. The results of hypotheses testing

Circle N°	t_y	f	$t_{1-\alpha;f}$	H	t_x	f	$t_{1-\alpha;f}$	H
1	1.0124	10	2.2281	H_o	0.7551	10	2.2281	H_o
2	0.1771	10	2.2281	H_o	0.4365	10	2.2281	H_o

According to obtained results it is possible to state that differences between circle's centers obtained by calculation based on the measurements and circle's centers determined on the base of points belonging to the circle line are equal in statistical sense. This fact allows the statement that it is possible to determine the center and radiuses of circles on the high level of reliability by geodetic methods in case when the points are regularly distributed on the circle and when they are determined from very short distance.

An interesting observation is that root mean square errors are quite small of hundredths of millimeter levels. This could be the consequence of more accurate measurements than it was declared measurement uncertainties or because of quite short distances between control and measured points on the circles. This finding deserves further research because it opens a possibility of utilizing geodetic methods for determining the dimensions of mechanical engineering elements at the high level of accuracy requirements.

4. CONCLUSION

The experiment in this research was designed to simulate the real situation in which could appear in geodetic professional practice for determination circle's center and radius. Two circles were drowned, printed, and put on the wall. After that the geodetic measurements were provided from two control points with measurements of directions, zenith angles and distances. After measurements the adjustment of measured values was provided, and circles' centers were determined. In the second stage the centers of circles were determined by utilizing the points belonging to circle and their equality was tested by the means of student's statistics. According to the results of statistical hypothesis testing it was proven the equality of the circles' centers. Also, the high accuracy of obtained results for circles' centers determination deserve further investigation.

LITERATURE

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