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OPTIMIZATION OF PURLINS CROSS-SECTION EXPOSED TO FIRE

Abstract

In practice, consideration of fire protection for structural elements mainly occurs after the adopted dimensions of sections. However, this procedure leads to not the most cost-effective solution in general. To find the optimal solution, it is necessary to apply one of the optimization methods. The presented optimization of purlins RHS cross-section is performed with nonlinear programming available in widely used program Excel. The objective function is defined as producing the purlin at a minimal price, considering the price of steel, work, and fire-resistant paint. Limits are introduced to ensure the cross-section satisfies the ultimate limit state for permanent and transient load situations, as well as in case of fire. Besides the ultimate limit states, the limits are defined for serviceability limit states and for cross-sectional geometry. Optimization analysis for different ISO 834 fire durations is followed by a result comparison. It provides an overview of cross-sectional parameters that most influence the bearing capacity in case of fire. It is concluded that by increasing the exposure time to fire, the optimal solution becomes a section with a smaller perimeter, larger surface area, and a thicker layer of fire-resistant coating.

Keywords: fire safety analysis, ISO 834, optimization, nonlinear programming

ОПТИМИЗАЦИЈА ПОПРЕЧНОГ ПРЕСЈЕКА РОЖЊАЧЕ ИЗЛОЖЕНЕ ПОЖАРУ

Сажетак

Разматрање заштите од пожара конструктивних елемената у пракси углавном долази након усвојених димензија пресјека. Међутим то у већини случајева није најисплативије рјешење. Да би се пронашло оптимално рјешење, потребно је примијенити неку од метода оптимизације. Приказана је оптимизација RHS попречног пресјека рожњаче употребом нелинеарног програмирања. Функција циља је дефинисана тако да се добије најмања цијена рожњаче, узимајући у обзир цијену челика, рада и противпожарног премаза. Ограничења су постављена тако да осигурају испуњење носивости при сталним и повременим прорачунским ситуацијама, као и при дејству пожара. Поред услова носивости, постављена су ограничења за гранично стање употребљивости и за геометрију попречног пресјека. Након извршених прорачуна, резултати за различито трајање стандардног ISO 834 пожара су упоређени. Поређење резултата оптимизације нам даје увид у карактеристике пресјека које највише утичу на пораст његове носивости при дејству пожара. На основу резултата анализе, закључено је да повећавајући вријеме изложености пожару, оптимално рјешење постаје пресјек који има мањи обим, а већу површину и дебљи слој противпожарног премаза.

Кључне ријечи: противпожарна анализа, ISO 834, оптимизација, нелинеарно програмирање

1. INTRODUCTION

In structure dimensioning, in most cases, the bearing capacities in permanent, transient, and seismic situations are considered. Meanwhile, the bearing capacity in a fire situation is given less importance. It is usually, in the case of steel structures, provided with fire-resisting materials [1]. In the case that necessary bearing capacity cannot be achieved by the chosen material, a cross-section with higher performances will be chosen [2, 3, 4].

To get the optimal cross-section, it is necessary to consider the bearing capacity in fire design situations in the process of dimensioning. To show the possibilities of an optimization algorithm applied to a structure that is exposed to fire, a brief review of previous studies follows. The study by Bendetti et al. [5] shows the optimization algorithm for steel I-section columns with and without fire-resistant coating. An optimization of steel moment frames is presented in a paper by Jarman et al. [6]. Hopkih et al [7] optimized the cross-section of a steel beam. The increase in constraints that must be satisfied during design, makes the optimization problem significantly more complex. An example is shown in a work by Albero et al. [8], who, in the process of finding the optimal solution, varied the size and shape of openings in hollow prestressed concrete slabs exposed to fire. A paper by Cauteren et al. [9] analyses the optimization of hybrid concrete-timber trusses. Besides the bearing capacity and serviceability in permanent and transient situations, bearing capacity in case of fire was analyzed, as well as their impact on the environment. Thai et al. [10] optimized a cross-section of composite CLT and concrete slabs, introducing the human-induced vibration check into the analysis, besides the usual constraints.

Previous studies, such as the work by Bendetti et al. [5] simplify the thermal analysis by disregarding the variation of material properties with temperature. Some studies present an optimization algorithm is not available in publicly available software [8, 9].

This paper shows a solution to an optimization problem, solved in Excel – widely used program from Microsoft, for a purlin of steel haul protected with a fire-resistant coating, which, with minor adjustments, can be used for other structural elements.

2. DESCRIPTION OF THE STRUCTURE AND ACTING LOADS NTRODUCTION

The considered hall structure is formed with main bearing frames at an intermediate distance of 5 m. The intermediate distances of purlins, which are bridging this span, are 2.5 m. Purlins static system is a continuous beam. The middle span will be analyzed. An insulated panel with 80 mm thickness is adopted as the roof cover. The roof slope is 6°, which classifies this roof as a duo pitch for wind and snow load analysis. The commonly applied rectangular hollow section (RHS) is adopted for the purlin (Figure 1).

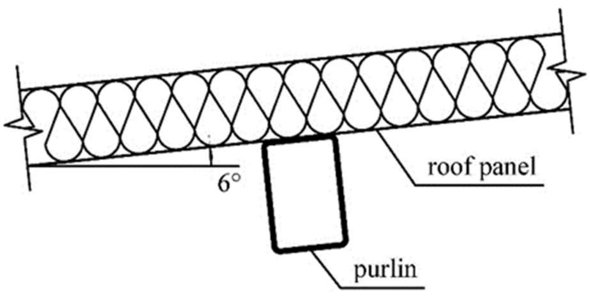

Figure 1. Analyzed purlin

The purlin is protected from heating with fire-resisting paint, that, when exposed to elevated temperatures, expands and forms an insulating layer around the structure. Taking into account that the insulated panels are placed on the top side of the purlin, that side of the cross-section is considered protected from heating.

The purlins are, besides their self-weight and panel weight, loaded with snow and wind. The following values are obtained from the load analysis:

- panels weight........................0.12 kN/m^2
- installations...........................0.30 kN/m^2
- snow......................................0.80 kN/m^2

- wind (pressure)....................0.30 kN/m^2
- wind (suction)......................-0.60 kN/m^2

Purlin self-weight is considered with its accurate value during optimization analysis.

3. CONSTRAINS

The conditions that must be fulfilled in optimization analysis are called constraints since they prevent the objective functions from having smaller or greater value. Depending on the nature of the constraint, it can be written in the form of an equality or an inequality relation.

Constraints that must be fulfilled in this analysis are the bearing capacity in permanent and transient situations, the serviceability limit state, the bearing capacity in a fire situation, and additional geometrical constraints.

3.1. BEARING CAPACITY IN PERMANENT AND TRANSIENT SITUATIONS

Before checking section bearing capacity, it is necessary to perform its classification according to SRPS EN 1993-1-1 [11]. Depending on the cross-sectional class, calculation can be performed by plasticity or elasticity theory. The bearing capacity of sections with classes 1 and 2 can be determined by the plasticity theory, while for sections with classes 3 and 4, the elasticity theory is applied. The bearing capacity of the cross-section of class 4 is performed with the effective cross-section.

The bearing capacity criterion in case of fire is very restrictive for the cross-sections with class 4. For that reason, these sections are not considered optimal, and therefore, are excluded from the study.

3.1.1. BEARING CAPACITY DETERMINATION BY PLASTICITY THEORY

When plasticity theory is used, the bearing capacity check is performed following SRPS EN 1993-1-1. The shear bearing capacity is checked per section 6.2.6 for both directions in the following way:

$$\frac{V_{Ed,\mathrm{y}}}{V_{\mathrm{pl,y,Rd}}} \leq 1.0 \qquad (1)$$

$$\frac{V_{Ed,\mathrm{z}}}{V_{\mathrm{pl,z,Rd}}} \leq 1.0 \qquad (2)$$

where:
V_{Ed} – shear force
$V_{\mathrm{pl,Rd}}$ – shear force resistance

The bending capacity check is done according to section 6.2.9:

$$\frac{\mathrm{M_{Ed,y}}}{\mathrm{M_{pl,y,Rd}}} + \frac{\mathrm{M_{Ed,z}}}{\mathrm{M_{pl,z,Rd}}} \leq 1.0 \qquad (3)$$

where:
$\mathrm{M_{Ed}}$ – bending moment
$\mathrm{M}_{pl,\mathrm{Rd}}$ – bending moment resistance

In the case that cross-sectional utilization for shearing is greater than 0.50, it is necessary to consider its interaction with the bending for capacity check (section 6.2.8).

3.1.2. BEARING CAPACITY DETERMINATION BY ELASTICITY THEORY

When the theory of elasticity is used, the shear capacity can be checked according to section 6.2.6. and expressions:

$$\tau_{Ed,\mathrm{y}} \leq \frac{f_y/\sqrt{3}}{\gamma_{M0}} \qquad (4)$$

$$\tau_{Ed,\mathrm{z}} \leq \frac{f_y/\sqrt{3}}{\gamma_{M0}} \qquad (5)$$

where:
τ_{Ed} – shear stress
f_y – yield strength of steel
γ_{M0} – partial safety factor

The bending bearing capacity check, according to the theory of elasticity, is performed per section 6.2.9 and the following expression needs to be satisfied:

$$\sigma_{x,\mathrm{Ed}} \leq \frac{f_y}{\gamma_{M0}} \tag{6}$$

where:
$\sigma_{x,\mathrm{Ed}}$ – normal stress from bending
In the case that shear bearing capacity, calculated by plasticity theory, is greater than 0.50, it is necessary to consider shear and bending interaction in the bearing capacity check per section 6.2.8.

3.2. SERVICEABILITY LIMIT STATE

A serviceability limit state check is necessary to control deflections in the middle of the purlins span. Deflection check, for continuous beam, can be performed by expression:

$$f = \sqrt{\left(\frac{q_{z,k}L^4}{384EI_y}\right)^2 + \left(\frac{q_{y,k}L^4}{384EI_z}\right)^2} \leq f_{limit} \tag{7}$$

where:
$q_{z,k}$ – load in z-axis direction, calculated for characteristic combination
$q_{y,k}$ – load in y-axis direction, calculated for characteristic combination
I_y – moment of inertia about y-axis
I_z – moment of inertia about z-axis
L – purlins span
E – elasticity modulus of steel
I – moment of inertia
f_{limit} – limit deflection
The limit deflection for purlins per SRPS EN 1993-1-1/NA [12] is L/200.

3.3. BEARING CAPACITY IN A FIRE SITUATION

A bearing capacity check in a fire situation is performed according to SRPS EN 1993-1-2 [13]. Expression for critical temperature $\theta_{a,cr}$ evaluation is given in section 4.2.4.:

$$\theta_{a,cr} = 39.19\, ln\left(\frac{1}{0.9674\mu_0^{3.833}} - 1\right) + 482 \tag{8}$$

where:
μ_0 – cross-sectional utilization for characteristic load combination for time t = 0
The cross-sectional utilization is calculated according to expressions provided in section 3.1. with rigorous criteria for cross-sectional classification provided in section 4.2.2.
The temperature of the insulated steel cross section $\Delta\theta_{a,t}$ is calculated according to section 4.2.5.2. and the expression:

$$\Delta\theta_{a,t} = \frac{\lambda_p A_p/V}{d_p c_a \rho_a}\frac{(\theta_{g,t}-\theta_{a,t})}{(1+\phi/3)}\Delta t - (e^{\frac{\phi}{10}} - 1)\Delta\theta_{g,t} \tag{9}$$

$$\phi = \frac{c_p \rho_p}{c_a \rho_a} d_p A_p/V \tag{10}$$

where:
A_p/V – cross-sectional factor
c_a – specific heat of steel
c_p – specific heat of insulating material
d_p – thickness of insulating material
Δt – time interval
$\theta_{g,t}$ – gas temperature
$\Delta\theta_{g,t}$ – increase in gas temperature
λ_p – thermal conductivity of insulating material
ρ_a – density of steel

ρ_p – density of insulating material

Intumescent coating significantly expands in a fire situation and in that way form the insulating layer around the protected element. The size of expansion mostly depends on the applied thickness of the intumescent coating. Modeling of intumescent coating with changing volume is complex and, for that reason, an approach is proposed for using the layer of initial thickness with equivalent thermal characteristics [14, 15, 16]:

$$c_p = 1200\, J/kgK \tag{11}$$

$$\lambda_p = -0.56 \times 10^2 + 11.8 d_p + 1.4 V/A_p \tag{12}$$

$$\rho_p = 200\, kg/m^3 \tag{13}$$

Gas temperature can be evaluated for standard ISO 834 fire per SRPS EN 1991-1-2 [17] and expression:

$$\theta_{g,t} = 20 + 345 \log{(8t + 1)} \tag{14}$$

where:

t – time

For the bearing capacity to be satisfactory it is necessary to satisfy the following condition:

$$\theta_{a,t} \leq \theta_{a,cr} \tag{15}$$

3.4. GEOMETRICAL CONSTRAINS

In order to exclude the cross-sections of class 4, it is necessary to limit the slenderness of cross-sectional parts. This is done following the cross-sectional classifications in fire situation. Constraints for width (b) and height (h) are formulated as a function of sectional thickness (t):

$$\frac{b-4t}{t} \leq 38\varepsilon \tag{16}$$

$$\frac{h-4t}{t} \leq 38\varepsilon \tag{17}$$

Coefficient ε can be calculated from the yield stress (f_y) according to section 4.2.2. and expression:

$$\varepsilon = 0.85\sqrt{235/f_y} \tag{18}$$

Besides constraints that are introduced for the cross-sectional class, the following constraints that limit the relative size of width and height are adopted:

$$\frac{h}{b} \leq 3 \tag{19}$$

4. OBJECTIVE FUNCTION

The objective function compares possible solutions that are calculated in iterations. Based on that comparison it is determined if the solution is optimal.

For the objective function, the price of purlins' one-meter length is adopted. The following prices are used in the calculation:

- steel with a montage: 2.5 $€/kg$
- intumescence paint: 85 $\frac{€}{2.5l}$
- paint application: 2 $€/m^2$

The adopted consumption of intumescence paint is 1.4 l/m^2 for a 1 mm layer.

Based on those prices and consumption, the objective function is specified as:

$$f_{min} = 2.5 V \rho_a + \left(1.4 \frac{85}{2.5} d_p + 2\right) A_p \tag{20}$$

where:

V – purlins volume

A_p – purlins area

5. OPTIMIZATION METHOD

The chosen method for optimization is a generalized reduced gradient which is implemented in Excel [18]. Its predecessor is the method of reduced gradient [19], in which constraints can only be defined in the form of equality. Extension of this method for inequality constraints is achieved through slack variables, similar to the simplex method in linear programming. The algorithm begins with the adoption of the initial solution $\vec{x}^{(0)}$ and the termination parameter $\varepsilon(\rightarrow 0^{+})$ that defines the acceptable error of the result. The next step is to transform all inequality constraints (type g) to equality (type h) with an introduction of slack variables. Values of y_i are calculated for all variables $x_i, i = 1,2,3 \dots N$, and they represent the relative distance of the current solution from the limits in which it can be located:

$$y_i = \frac{min\{x_i - x_i{}^L, x_i{}^U - x_i\}}{x_i{}^U - x_i{}^L} \tag{21}$$

where:
$x_i{}^L$ – lower limit of feasible solutions x_i
$x_i{}^U$ – upper limit of feasible solutions x_i
Evaluation of variables y_i is followed by their sorting in a descending order. Variables x_i are chosen for basic variables if they have a higher y_i value. The basic variable quantity is equal to the number of equality constraints. The rest of x_i variables are non-basic and their values are determined from constraints, i.e., they are equal to the slack variables. The gradients of the objective function are now determined. Following that the reduced gradient of the objective function $\nabla\tilde{f}(\vec{x}^{(t)})$ is evaluated as:

$$\nabla\tilde{f}(\vec{x}^{(t)}) = \nabla\bar{f}(\vec{x}^{(t)}) - \nabla\hat{f}(\vec{x}^{(t)})J^{-1}.C \tag{22}$$

where:
$\nabla\bar{f}(\vec{x}^{(t)})$ – component of objective function gradient that is composed of basic variables
$\nabla\hat{f}(\vec{x}^{(t)})$ – component of objective function gradient that is composed of non-basic variables
J – component of constraints gradient that is composed of basic variables
C – component of constraints gradient that is composed of non-basic variables
It is checked whether the condition for stopping the calculation is fulfilled, i.e., whether the current solution is close enough to the optimal one:

$$\left\|\nabla\tilde{f}(\vec{x}^{(t)})\right\| < \varepsilon \tag{23}$$

If the previous condition is satisfied, the current solution is considered as final. In case the condition is not met, the direction in which the solution is sought in the next iteration is determined. It consists of two components, the first consisting of non-basic variables, and the second of basic ones. The component with non-basic variables $\bar{d}$ can be determined in the following way:

$$\bar{d} = \begin{cases} 0, if\ x_i = x_i{}^L \text{ и } (\nabla\tilde{f})_i > 0 \\ 0, if\ x_i = x_i{}^U \text{ и } (\nabla\tilde{f})_i < 0 \\ -(\nabla\tilde{f})_i, otherwise \end{cases} \tag{24}$$

while we determine the component of basic variables $\hat{d}$ according to the expression:

$$\hat{d} = -J^{-1}C\bar{d} \tag{25}$$

After that, the parameter $\alpha^{(t)}$ is determined, for which the objective function in the form $f(x^t + \alpha^{(t)}d)$ reaches a minimum. This parameter dictates how far one goes in the direction of d in which the solution is sought for the next iteration. The solution for the next iteration is adopted in the form:

$$x^{(t+1)} = x^t + \alpha^{(t)}d \tag{26}$$

The counter indicates that the iteration of the algorithm is increased by one, and the process returns to the beginning:

$$t = t + 1 \tag{26}$$

The method's limitation lies in the fact that the gradient of the function equals zero not only at global minimum, but also at local minimums, making the solution reliant on the initial solution (Figure 2).

To avoid the problem of local minimum, a set of initial solutions is adopted for variable values. The initial solutions are adopted randomly within predefined domains, ensuring the highest degree of exploration of the space of possible solutions and the greatest chance of reaching the global minimum. Depending on the analyzed problem, i.e., the number of local minimum that exist, it is necessary to adopt a different quantity of initial solutions. A larger number of initial solutions significantly extends the calculation time, so it is essential to conduct a sensitivity analysis of the final solution based on the adopted number of initial solutions. This has been done in this study, and it has been concluded that, for the analyzed optimization problem, it can be confidently assumed that reaching the global minimum is achievable with one hundred initial solutions.

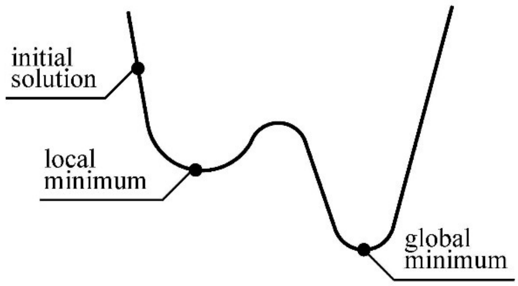

Figure 2. Local minimum problem

6. RESULT COMPARISON

The calculation is carried out for situations without fire and for standard fire durations of 30, 60, 90, and 120 minutes. In the determination of optimal solution, values are varied for: width, height, and thickness of the cross-section, as well as thickness of intumescence paint. The optimization results are shown in Table 1.

Based on the results it can be concluded that for the greater fire duration, a thicker layer of intumescence paint, a thicker cross-section, and a smaller cross-sectional factor are necessary. The thicker cross-section is needed because the thickness limit of intumescence paint, which is cheaper than steel, is achieved and because the thicker cross-section is heating slower. Because of the requirement for a thicker cross-section, its width and height are contracted. In this way, the cross-section has a minimal price and fulfills all constraints for fire situations and permanent and transient situations.

Table 1. Optimization results

variable	t = 0	R30	R60	R90	R120
b [mm]	79.66	79.66	72.55	71.94	57.26
h [mm]	116.24	116.24	118.08	109.99	75.74
t [mm]	3.30	3.30	3.78	5.38	10.66
d_p [mm]	0	0	0.73	1.92	3.00
f_{min} [€/m]	25.29	25.29	29.27	40.22	51.06

7. CONCLUSION

The paper analyses an optimization problem of a purlins cross-section exposed to fire, that is protected with intumescence paint. Constraints that ensure the bearing capacity and serviceability checks are verified in permanent and transient situations, as well as in fire cases are imposed. To formulate the objective function, the price of steel, montage, and intumescence paint are considered. The generalized reduced gradient is adopted for the optimization method. For the effective use of the chosen optimization method, it was necessary to assume a set of initial solutions, which produce different results. The result that corresponds to the minimum value of the objective function is considered as the global minimum.

The presented optimization analysis performed in Excel, in the considered case, enables the determination of a cross-section that fulfills all constrains concerning bearing capacity, serviceability, and geometry and has a minimal cost. Besides that, it provides insight into the behavior of the elements in fire situations, and on a specific example shows the effect of cross-sectional factors on the fire resistance.

Although the paper only analyses an example of a purlin cross-section exposed to fire, the same optimization analysis can successfully solve other complex problems with several constraints. The ongoing study extends the algorithm to the optimization of other structural elements.

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