70
Research paper
Doi 10.61892/stp202401040M
ISSN 2566-4484

Snježana Maksimović, University of Banja Luka , snjezana.maksimovic@aggf.unibl.org
Arslan Ansari, Islamic Azad University, Karaj Branch, analsisamirmath2@gmail.com
Sandra Kosić-Jeremić, University of Banja Luka, sandra.kosic-jeremic@aggf.unibl.org

NESTED POLYNOMIALS TRIGONOMETRIC AND HYPERBOLIC TYPE WITH APPLICATIONS

Abstract

In this paper we introduce nested trigonometric and hyperbolic polynomials of rank n by employing nested trigonometric and hyperbolic functions, respectively. Our investigation delves into some interesting properties of nested polynomials, establishing some recurrence formulas and deriving some useful summation formulas. Additionally, we establish a connection between nested trigonometric polynomials and one specific class of special polynomials that satisfy a second-order Sturm-Liouville differential equation. Finally, we outline several applications of nested polynomials. Among others, it is shown that nested trigonometric polynomials can be used for the linear static analysis of curved Bernoulli–Euler beam.

Keywords: Nested functions, Nested polynomials, Mittag-Leffler functions, Sturm-Liouville differential equation

ТРИГОНОМЕТРИЈСКИ И ХИПЕРБОЛИЧКИ УГЊЕЖДЕНИ ПОЛИНОМИ СА ПРИМЈЕНАМА

Сажетак

У раду уводимо угњеждене тригонометријске и хиперболичне полиноме ранга н користећи угњеждене тригонометријске и хиперболичке функције, редом. Истражујемо нека њихова занимљива својства, успостављајући неколико рекурзивних формула и изводећи неке корисне сумацијске формуле. Додатно, успостављамо везу између угњеждених тригонометријских полинома и једне специфичне класе специјалних полинома који задовољавају Штурм-Лиувилову диференцијалну једначину другог реда. На крају, наводимо неколико примјена уведених полинома. Између осталог, показано је да се тригонометријски полиноми могу користито за линеарну статичку анализи закривљене Ојлер-Бернулијеве греде.

Кључне ријечи: угњеждене функције, угњеждени полиноми, Митаг-Лефлерове функције, Штурм-Лиувилова диференцијална једначина

1. INTRODUCTION

The Mittag-Leffler function, introduced by Swedish mathematician Gösta Magnus Mittag-Lefler in [1,2], is a special function of a particular form

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}, \alpha, z \in C, Re(\alpha) > 0,$$

where Γ is the gamma function. This function is presented in relation to his approach for summing certain divergent series. During the first half of the twentieth century, the Mittag-Leffler function remained almost unknown to the majority of scientists. Since then, Mittag-Leffler functions have been studied extensively, and they play a crucial role in understanding complex systems with memory effects and fractional-order dynamics. So, with the development of fractional calculus, scientists' interest in Mittag-Leffler functions has surged [3-6]. Solutions to certain linear fractional differential equations can be expressed using Mittag-Leffler functions [7, 8]. Over the past two decades, the appeal of Mittag-Leffler functions and Mittag-Leffler type functions has notably increased, particularly among engineers and scientists, owing to their extensive application in various practical problems, including fluid flow, diffusion-like transport, electrical networks, probability and statistical distribution theory [9-11]. Of particular interest are the trigonometric and hyperbolic type functions based on the Mittag-Leffler function introduced in [12]. Also, these functions are called „nested", because with the finite derivative they become functions in the same class.

This paper introduces nested trigonometric and hyperbolic type polynomials by employing nested trigonometric and hyperbolic functions. In Section 3 of the paper, we explored several properties of nested polynomials, established recurrence and summation formulas. The subsequent part of the paper, Section 4, focuses on applying these introduced polynomials.

2. PRELIMINARIES AND NOTATATION

In this section we recall some definitions and assertions from [12-14].

2.1. TRIGONOMETRIC AND HYPERPOBLIC TYPE FUNCTIONS

In [12] functions of trigonometric type with p elements $T_{pj}: R \to R, j = 0,1, \dots, p-1, p \in N$, are defined as

$$T_{pj}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{pn+}}{(pn+j)!}.$$

They can be named *p tuple nested function of trigonometric type* because after some step derivatives return to itself, that is

$$T'_{p0}(x) = -T_{pp-1}(x), \quad T'_{p1}(x) = T_{p0}(x), \quad \dots, \quad T'_{pp-1}(x) = T_{pp-2}(x).$$

For example,

$$T_{10}(x) = e^{-x}, \quad T_{20}(x) = \cos x, \quad T_{21}(x) = \sin x.$$

Functions of hyperbolic type with p elements $H_{pj}: R \to R, j = 0,1, \dots, p-1, p \in N$, are defined as [12]

$$H_{pj}(x) = \sum_{n=0}^{\infty} \frac{x^{pn+j}}{(pn+j)!}.$$

They can be named *p tuple nested function of hyperbolic type* because after some step derivatives return to itself, that is

$$H'_{p0}(x) = H_{pp-1}(x), \quad H'_{p1}(x) = H_{p0}(x), \quad \dots, \quad H'_{pp-1}(x) = H_{pp-2}(x).$$

For example,

$$H_{10}(x) = e^x, \quad H_{20}(x) = \cosh x, \quad H_{21}(x) = \sinh x.$$

Functions T_{pj} and H_{pj} find extensive applications across various domains as they serve as solutions to certain classes of differential equations and fractional differential equations [15, 16]. For example, the solution of the differential equation [15]

$$y'''(x) - y(x) - 1 = 0, \qquad y(0) = 1, \qquad y'(0) = 2, \qquad y''(0) = 4$$

is

$$y(x) = 2H_{30}(x) + 2H_{31}(x) + 4H_{32}(x) - 1.$$

Also, nested functions are solutions of some classes of partial differential equations which in the boundary and initial conditions contain triple nested functions of trigonometric and of hyperbolic type [17].

2.2. SPECIAL POLYNOMIALS AND SPECIAL FUNCTIONS

In [13] are studied the special polynomials of the form

$$F_0(x) = 1, F_{2n}(x) = \sum_{k=0}^{n} (-1)^{n+k} \binom{2n}{2k} x^{2k}, F_{2n-1}(x) = \sum_{k=1}^{n} (-1)^{n+k+1} \binom{2n}{2k-1} x^{2k-1} \qquad (1)$$

Although they are non-orthogonal, they play important role because are enumerators of rational functions of the form

$$f_0(x) = 1, \qquad f_{2n}(x) = \frac{(-1)^n F_{2n}(x)}{(x^2+1)^n}, \quad f_{2n-1}(x) = \frac{(-1)^{n-1} F_{2n-1}(x)}{(x^2+1)^n}, n \in N, \qquad (2)$$

which have several applications. For example, these functions can be used for the arc-length parameterized curves of the form [14]

$$r_{2n}(s) = \langle \int_0^s f_{2n}(x) dx, \int_0^s f_{2n-1}(x) dx \rangle, \qquad (3)$$

where $s > 0$ is the length of the curve. The arc-length curves are useful in engineering applications, especially for the analysis of beam-like structures. Curves (3) are employed in the linear static analysis of curved Bernoulli-Euler beams because the beam equations possess analytical solutions, which is a rarity in this field [14].

Theorem 1.1. [13] For special polynomials $F_n(x)$, the following summation formulas hold

$$e^x = \frac{1}{\cos 1} \sum_{n=0}^{\infty} \frac{F_{2n}(x)}{(2n)!} - \frac{1}{\sin 1} \sum_{n=1}^{\infty} \frac{F_{2n-1}(x)}{(2n)!},$$

$$\cos(x) = \frac{1}{\cosh 1} \sum_{n=0}^{\infty} (-1)^n \frac{F_{2n}(x)}{(2n)!}, \qquad \sin(x) = \frac{1}{\sinh 1} \sum_{n=1}^{\infty} (-1)^n \frac{F_{2n-1}(x)}{(2n)!},$$

$$\cosh(x) = \frac{1}{\cos 1} \sum_{n=0}^{\infty} \frac{F_{2n}(x)}{(2n)!}, \qquad \sinh(x) = -\frac{1}{\sinh 1} \sum_{n=1}^{\infty} \frac{F_{2n-1}(x)}{(2n)!}.$$

Theorem 1.2.[18] For special polynomials $F_n(x)$, the following recurrence formulas hold

$$F'_{2n+2}(x) = 2(n+1)\big(xF_{2n}(x) + F_{2n-1}(x)\big),$$

$$F_{2n+2}(x) = (x^2-1)F_{2n}(x) + 2xF_{2n-1}(x),$$

$$xF'_{2n}(x) = n((x^2+1)F_{2n-2}(x) + F_{2n}(x)).$$

Theorem 1.3. [14] For polynomials $F_n(x), n \in N$, the following hold

$$F^2_{2n-1}(x) + F^2_{2n}(x) = (x^2+1)^{2n}.$$

Theorem 1.4. [14] Polynomials $F_n(x), n \in N_0$, are solutions of the Sturm-Liouvile differential equation

$$(x^2+1)y''(x) - 2(2n-1)xy'(x) + 2n(2n-1)y(x) = 0. \qquad (4)$$

Moreover, $y_n(x) = C_1 F_{2n}(x) + C_2 F_{2n-1}(x)$ are the only solutions of (4).

3. NESTED TRIGONOMETRIC AND HYPERBOLIC POLINOMIALS

In this section we introduce nested trigonometric and hyperbolic polynomials of rank n.

Definition 2.1. Let $T_{pnj}: R \to R,\ j = 0,1,\dots,p-1,\ p,n \in N_0$. We say that T_{pnj} is a polynomial of trigonometric type of rank n, with p element, if

$$T_{p00}(x) = 1, T_{pn0}(x) = \sum_{k=0}^{n} (-1)^k \binom{pn}{pk} x^{pk},$$

$$T_{pnj}(x) = \sum_{k=0}^{n-1} (-1)^k \binom{pn}{pk+j} x^{pk+j}, j = 1,\dots,p-1.$$

For p=2, we have

$$T_{200}(x) = 1, T_{2n0}(x) = \sum_{k=0}^{n} (-1)^k \binom{2n}{2k} x^{2k}, T_{2n1}(x) = \sum_{k=0}^{n-1} (-1)^k \binom{2n}{2k+1} x^{2k+1}, n \in N.$$

Theorem 2.1. If $\lambda^3 = -1, \lambda \in R, \lambda \neq -1$, then for trigonometric polynomials $T_{3nj}(x)$, $j = 0,1,2, n \in N$,

$$T_{3n0}(x) + \lambda T_{3n1}(x) + \lambda^2 T_{3n2}(x) = (\lambda x + 1)^{3n}$$

hold.

Proof. From the binomial formula, we obtain

$$(\lambda x + 1)^{3n} = \sum_{i=0}^{3n} \binom{3n}{i} (\lambda x)^i$$

$$= \sum_{k=0}^{n} \binom{3n}{3k} x^{3k} \lambda^{3k} + \sum_{k=0}^{n-1} \binom{3n}{3k+1} x^{3k+1} \lambda^{3k+1} + \sum_{k=0}^{n-1} \binom{3n}{3k+2} x^{3k+2} \lambda^{3k+2}.$$

Since $\lambda^3 = -1$, we have

$$(\lambda x + 1)^{3n} = \sum_{k=0}^{n} (-1)^k \binom{3n}{3k} x^{3k} + \lambda \sum_{k=0}^{n-1} (-1)^k \binom{3n}{3k+1} x^{3k+1} + \lambda^2 \sum_{k=0}^{n-1} (-1)^k \binom{3n}{3k+2} x^{3k+2},$$

which proves the assertion.

From Theorem 2.1 we immediately obtain the next assertion.

Theorem 2.2. If $\lambda^p = -1, \lambda \in R, \lambda \neq -1$, then for trigonometric polynomials $T_{pnj}(x)$, $j = 0,1,\dots,p-1, p > 3, n \in N$,

$$\sum_{j=0}^{p-1} \lambda^j T_{pnj}(x) = (\lambda x + 1)^{pn}$$

hold.

Theorem 2.3. If $\lambda^p = -1, \lambda \in R, \lambda \neq -1$, then for hyperbolic polynomials $T_{pnj}(x)$, $j = 0,1,\dots,p-1, p > 3, n \in N$,

$$(-1)^n (T_{pn}\ (x) - \sum_{j=1}^{p-1} \lambda^{p-j} T_{pnj}(x)) = (x + \lambda)^{pn}$$

hold.

Proof. From the binomial formula and $\lambda^p = -1$ we have

$$(x + \lambda)^{pn} = \sum_{k=0}^{pn} \binom{pn}{k} x^k \lambda^{pn-k}$$

$$= \sum_{k=0}^{n} \binom{pn}{pk} x^{pk} \lambda^{p(n-k)} + \sum_{k=0}^{n-1} \binom{pn}{pk+1} x^{pk+1} \lambda^{p(n-k-1)+p-1} + \cdots$$

$$+ \sum_{k=0}^{n-1} \binom{pn}{pk+p-1} x^{pk+p-} \ \lambda^{p(n-k-1)+1}$$

$$= \sum_{k=0}^{n} (-1)^{n-k} \binom{pn}{pk} x^{pk} + \lambda^{p-1} \sum_{k=0}^{n-1} (-1)^{n-k-1} \binom{pn}{pk+1} x^{pk+1} + \cdots$$

$$+ \lambda \sum_{k=0}^{n-1} (-1)^{n-k-1} \binom{pn}{pk+p-1} x^{pk+p-1}.$$

So, we obtain the assertion.

Definition 2.2. Let $H_{pnj}: R \to R,\ j = 0,1, \ldots, p-1, p, n \in N_0$. We say that H_{pnj} is a polynomial hyperbolic type of rank n, with p element, if

$$H_{p00}(x) = 1, H_{pn0}(x) = \sum_{k=0}^{n} \binom{pn}{pk} x^{pk}, \quad H_{pnj}(x) = \sum_{k=0}^{n-1} \binom{pn}{pk+j} x^{pk+j}, j = 1, .., p-1.$$

For p=2, we have

$$H_{200}(x) = 1, H_{2n0}(x) = \sum_{k=0}^{n} \binom{2n}{2k} x^{2k}, H_{2n1}(x) = \sum_{k=0}^{n-1} \binom{2n}{2k+1} x^{2k+1}, n \in N.$$

From the binomial formula, we obtain

$$H_{2n0}(x) + H_{2n1}(x) = \sum_{k=0}^{n} \binom{2n}{k} x^{2n-k} = (x+1)^{2n}.$$

So, in general case we have

$$H_{pn0}(x) + H_{pn}\ (x) + \cdots + H_{pnp-1}(x) = (x+1)^{pn}.$$

Theorem 2.4. If $\omega^3 = 1, \omega \in R, \omega \neq 1$, then for hyperbolic polynomials $H_{3nj}(x),\ j = 0,1,2, n \in N$,

$$H_{3n0}(x) + \lambda H_{3n1}(x) + \lambda^2 H_{3n2}(x) = (\omega x + 1)^{3n}$$

hold.

From Theorem 2.4 we immediately obtain the next assertion.

Theorem 2.5. If $\omega^p = 1, \omega \in R, \omega \neq 1$, then for hyperbolic polynomials $H_{pnj}(x),\ j = 0,1, \ldots, p-1, p > 3, n \in N$,

$$\sum_{j=0}^{p-1} \omega^j H_{pnj}(x) = (\omega x + 1)^{pn}$$

hold.

Theorem 2.6. If $\omega^p = 1, \omega \in R, \omega \neq 1$, then for hyperbolic polynomials $H_{pnj}(x),\ j = 0,1, \ldots, p-1, p > 3, n \in N$,

$$\sum_{j=0}^{p-1} \omega^{p-j} H_{pnj}(x) = (x + \omega)^{pn}$$

hold.

Proof. Using the binomial formula, we have

$$(x + \omega)^{pn} = \sum_{i=0}^{pn} \binom{pn}{i} x^i \omega^{pn-i}$$

$$= \sum_{k=0}^{n} \binom{pn}{pk} x^{pk} \omega^{p(n-k)} + \sum_{k=0}^{n-1} \binom{pn}{pk+1} x^{pk+1} \omega^{p(n-k-1)+p-1} + \cdots$$
$$+ \sum_{k=0}^{n-1} \binom{pn}{pk+p-1} x^{pk+p-} \ \omega^{p(n-k-1)+1}.$$

Since $\omega^p = 1$, we have

$$(x+\omega)^{pn} = \sum_{k=0}^{n} \binom{pn}{pk} x^{pk} + \omega^{p-1} \sum_{k=0}^{n-1} \binom{pn}{pk+1} x^{pk+1} + \cdots + \omega \sum_{k=0}^{n-1} \binom{pn}{pk+p-1} x^{pk+p-1},$$

which proves the assertion.

3.1. CONNECTION BETWEEN SPECIAL POLYNOMIALS AND TRIGONOMETRIC POLYNOMIALS

Using Definition 2.1 and (1) we obtain the connection between special polynomials $F_n(x)$ and polynomials $T_{2nj}(x), j = 0,1$:

$$F_0(x) = T_{200}(x), \quad F_{2n}(x) = (-1)^n T_{2n0}(x), \quad F_{2n-1}(x) = (-1)^n T_{2n1}(x), \ n \in N. \tag{5}$$

Theorem 2.7. Polynomials $T_{2nj}(x), j = 0,1, n \in N$, satisfy

$$T_{2n0}^2(x) + T_{2n1}^2(x) = (x^2+1)^{2n}.$$

Proof. Using (5) and Theorem 1.3 we obtain the proof.

Theorem 2.8. For polynomials $T_{2nj}(x), j = 0,1, n \in N$, it holds that

$$T'_{2(n+1)0}(x) = -2(n+1)(xT_{2n0}(x) + T_{2n1}(x)),$$
$$xT'_{2(n+1)0}(x) = (n+1)(T_{2(n+1)0}(x) - (x^2+1)T_{2n0}(x)),$$
$$T_{2(n+1)0}(x) = (1-x^2)T_{2n0}(x) - 2xT_{2n1}(x).$$

Proof. Using Theorem 1.2 and (5) we obtain the assertion.

Theorem 2.9. For nested functions and nested polynomials, the following summation formulas

$$H_{10}(x) = \frac{1}{\cos 1} \sum_{n=0}^{\infty} (-1)^n \frac{T_{2n0}(x)}{(2n)!} - \frac{1}{\sin 1} \sum_{n=1}^{\infty} (-1)^n \frac{T_{2n1}(x)}{(2n)!}$$
$$T_{20}(x) = \frac{1}{\cosh 1} \sum_{n=0}^{\infty} \frac{T_{2n0}(x)}{(2n)!}, T_{21}(x) = \frac{1}{\sinh 1} \sum_{n=1}^{\infty} \frac{T_{2n1}(x)}{(2n)!}$$
$$H_{20}(x) = \frac{1}{\cos 1} \sum_{n=0}^{\infty} (-1)^n \frac{T_{2n0}(x)}{n!}, H_{21}(x) = -\frac{1}{\sin 1} \sum_{n=1}^{\infty} (-1)^n \frac{T_{2n1}(x)}{n!}$$

hold.

Proof. Follows from Theorem 1.1.

4. APPLICATIONS

Theorem 3.1. Polynomials $T_{2n}\ (x), j = 0,1, n \in N$, are solutions of the Sturm-Liouville differential equation (4). Moreover, only $y_n(x) = CT_{2n0}(x) + DT_{2n1}(x)$ are the solutions of (4).

Proof. The proof follows from Theorem 1.4.

Using (2) and (5) we obtain

$$f_0(x) = 1, f_{2n}(x) = \frac{T_{2n0}(x)}{(x^2+1)^n} = \cos(2n\ arctan(x)),$$
$$f_{2n-1}(x) = -\frac{T_{2n1}(x)}{(x^2+1)^n} = \sin(2n\ arctan(x)), n \in N.$$

It is proved in [14] that functions (2) can be used for the linear static analysis of curved Bernoulli–Euler beam, where the beam axis is described with

$$x(s) = \ln(1+s^2)\,, y(s) = 2\arctan(s) - s, s \in [0, s_L],$$

where s_L is the solution of the equation $y(s_L) = 0$. So, trigonometric polynomials $T_{2nj}(x), j = 0,1, n \in N,$ can be used for the linear static analysis of curved Bernoulli–Euler beam.
The equilibrium differential equations for a curved Bernoulli–Euler beam seldom possess analytical solutions. Nonetheless, employing arc-length parameterized curves (3) enables the discovery of precise solutions for linear static analysis of curved beams. The availability of analytical solutions in computational mechanics is greatly advantageous, serving as crucial reference benchmarks for evaluating new mechanical models and numerical techniques.

5. CONCLUSION

The Mittag-Leffler function, as a generalization of the exponential function plays a significant role in fractional calculus and has applications in various areas of mathematics and physics. This function arises naturally in problems involving fractional calculus, fractional differential equations, and in the context of anomalous diffusion and relaxation phenomena.
In this paper, we focused on two classes of Mittag-Leffler functions, called nested trigonometric and hyperbolic. Utilizing these functions, we formed nested polynomials of type trigonometric and type hyperbolic that we connected with special polynomials introduced in [13]. We have examined certain properties of these polynomials, obtained recursive relations and summation formulas. Using the connection (5) between nested trigonometric polynomials and the polynomials introduced in [13], we have shown that these polynomials are solutions to the Sturm-Liouville differential equation. Additionally, their application has been demonstrated, since trigonometric polynomials can be used for the linear static analysis of curved Bernoulli–Euler beam.

Acknowledgment
This work is supported by the Project 19.032/052-868/24. of the Republic of Srpska Ministry for Scientific and Technological Development and Higher Education.

LITERATURE

[1] G. M. Mittag-Leffler, “Une generalisation de l'integrale de Laplace-Abel”, Comptes Rendus de *l'Academie des Sciences Serie II*, vol. 137, pp. 537-539, 1903.
[2] G. M. Mittag-Leffler, “Sur la nouvelle fonction Eα(z) ”, *Comptes Rendus de l'Academie des Sciences*, vol. 137, pp. 554-558, 1903.
[3] R. Goreno, A.A. Kilbas, F. Mainardi, S.V. Rogosin, “Mittag-Leffler Functions, related topics and applications”, vol. 2. Springer, Berlin, 2014.
[4] R. Goreno, A. A. Kilbas, F.Mainardi, S. Rogosin, “Mittag-Leffler functions, related topics and applications”, Second Edition, Springer-Verlag GmbH Germany, part of Springer Nature 2020.
[5] H.J. Haubold, A.M. Mathai, R.K. Saxena, “Mittag-Leffler functions and their applications*”, J. Appl. Math*., 1-51, 2011.
[6] P. Humbert, “Quelques resultats relatifs_a la fonction de Mittag-Leffler”, *C. R. Hebd. Seances Acad. Sci.,* 236(15), 1467-1468, 1953.
[7] R.L. Bagley, “The initial value problem for fractional order differential equations with constant coeffcients”, Air Force Institute of Technology Report, AFIT-TR-EN-88-1, 1988.
[8] G. Dattoli, K. Gorska, A. Horzela, S. Licciardi, R.M. Pidatella, “Comments on the properties of Mittag-Leffler function”, *Eur. Phys. J. Spec. Top*. 226(16-18), 3427-3443, 2017.
[9] A. Erdelyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, “Higher transcendental functions”, vol. 3, McGraw-Hill, New York, NY, 1955.
[10] Y.L. Luke, “Special functions and their approximations”, Vol. 1, Ed. Academic Press, 1969.
[11] X.J. Yang, “Theory and applications of special functions for scientists and engineers”, Springer; 1st ed. , 2021.
[12] A. H. Ansari , X.L. Liu,V. N. Mishra, “On Mittag-Leffler function and beyond”, Nonlinear Sci. Lett. A, 8(2),187-199, 2017.
[13] N. Djurić, S. Maksimović, S. Gajić, “Summation formulas using a new class of special polynomials”. In Proceedings of the 19th International Symposium Infoteh-Jahorina, East Sarajevo, Bosnia and Herzegovina, 18–20 March 2020; pp. 1–4.

[14] S. Maksimović, A. Borković, “A New Class of Plane Curves with Arc Length Parametrization and Its Application to Linear Analysis of Curved Beams”, *Mathematics*, 9(15),1778, 2021.

[15] S. Maksimović, A.H. Ansari, “Solving conformable fractional Sturm-Liouville equations using one class of special polynomials and special functions, *MACA* 5 (1), 69-84, 2023.

[16] A. H. Ansari, S. Maksimović, H. A. Nabwey, Z. D. Mitrović, “Solving some classes of conformable fractional differential equations by the fractional Laplace transform method via some special classes functions of the Mittag-Leffler type”, preprint.

[17] A. H. Ansari , A. K. Sedeeg, S. Maksimović, “Double integral transform (Laplace-ARA transform) with triple nested functions of type trigonometry and type hyperbolic”, preprint.

[18] S. Maksimović, N. Djurić, I. V. Boroja, S. Kosić-Jeremić, “Some recurrence formulas for a new class of special polynomials and special functions”, International Conference on contemporary theory in construction XIV, AGGF Banja Luka, 2020, 71-76.